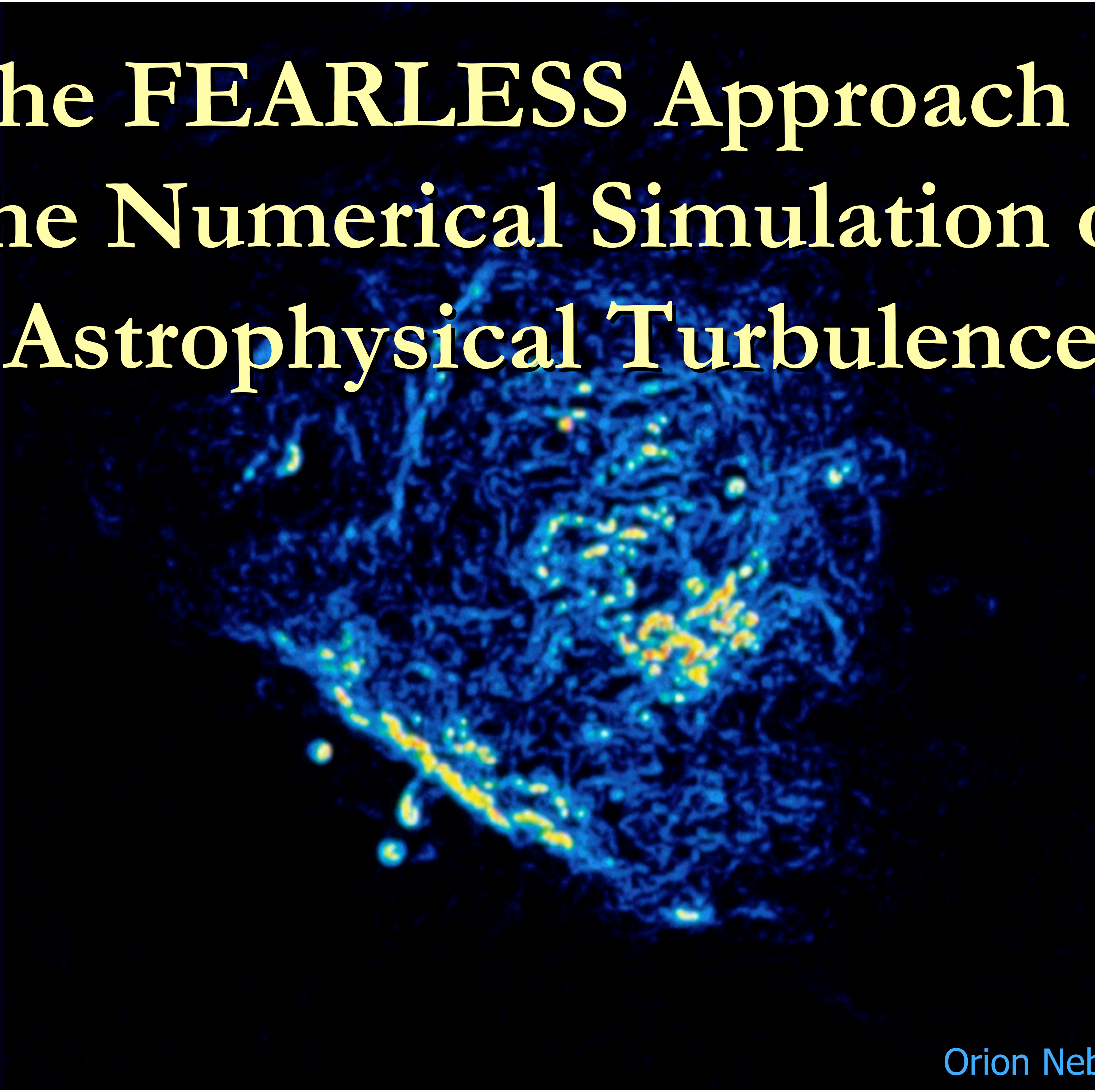
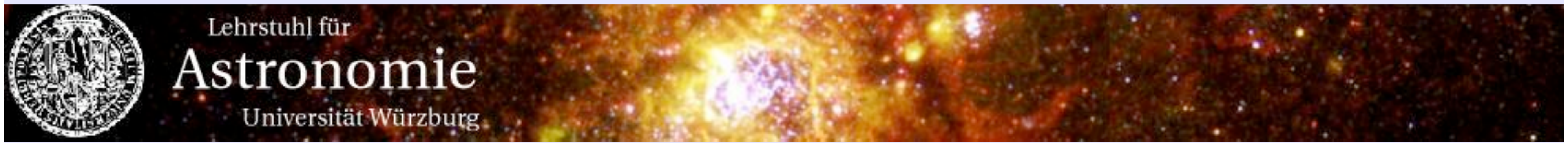
The FEARLESS Approach to the Numerical Simulation of Astrophysical Turbulence



Orion Nebula (VLA)

Wolfram Schmidt

J. C. Niemeyer, L. Iapichino
A. Maier, M. Hupp, Ch. Federrath



In collaboration with

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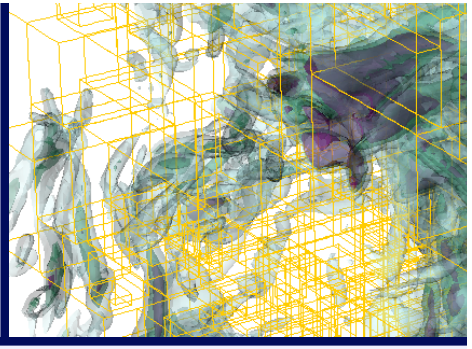
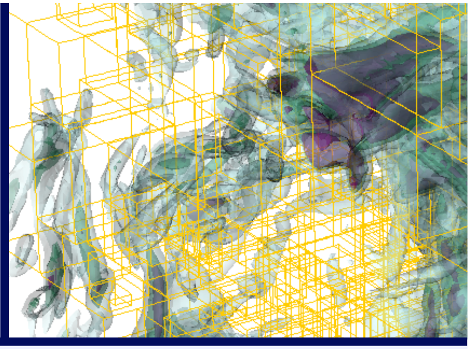
A. Kritsuk (Laboratory f. Comp. Astrophysics, UCSD)

Motivation

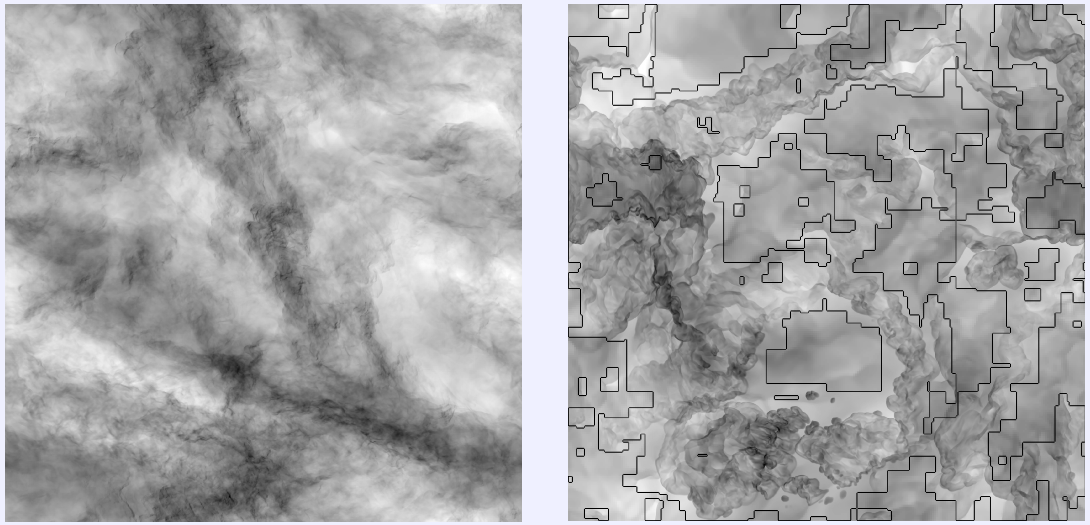


- There is evidence that galactic star formation is controlled by the turbulent interstellar medium (ISM)
- At length and time scales comparable to the size and life time of giant molecular clouds, turbulence in the ISM is transient and inhomogeneous
- This poses a numerical challenge:
  - Large eddy simulation (LES) is successful in treating homogeneous turbulence
  - SPH, on the other hand, is suitable for self-gravitating gas, but turbulent flow tends to be elusive

A Possible Solution

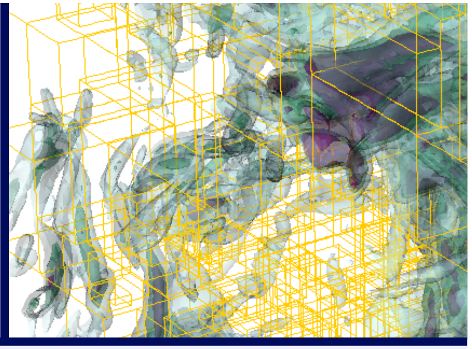


- Adaptive mesh refinement (AMR) offers flexibility comparable SPH
- The finit-volume approach employed with AMR allows for controlable dissipation properties and a well defined cutoff length



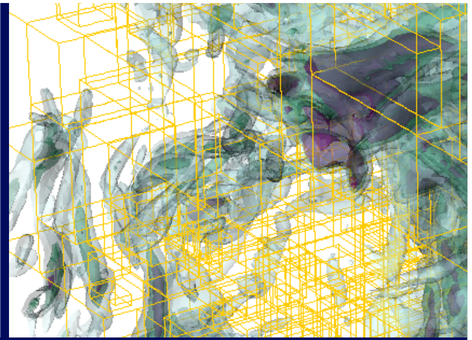
Kritsuk et al. (2006)

But Turbulence is Space-Filling, Right?



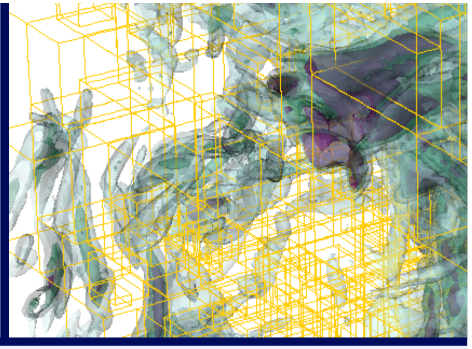
- Homogeneous turbulence is space-filling from the view point of the ensemble average (Kolmogorov theory, $E(k) \sim k^{-5/3}$)
- However, turbulence is intermittent
- At any instant of time, dissipative structures (turbulent eddies) are concentrated in regions of fractal dimension D less than 3
- Boldyrev et al. (2002) picked the estimate $D \approx 2.3$ (Elmegreen & Falgarone, 1996) and found $E(k) \sim k^{-1.83}$ with the β-model

A Few Questions…



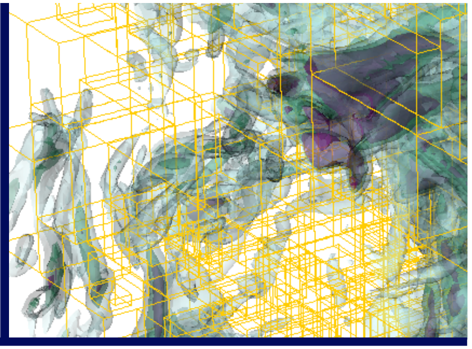
- Are turbulence simulations with AMR feasible?
  - What are appropriate refinement criteria?
    - How do subgrid scale effects come in?

Simulations of Forced Supersonic Turbulence

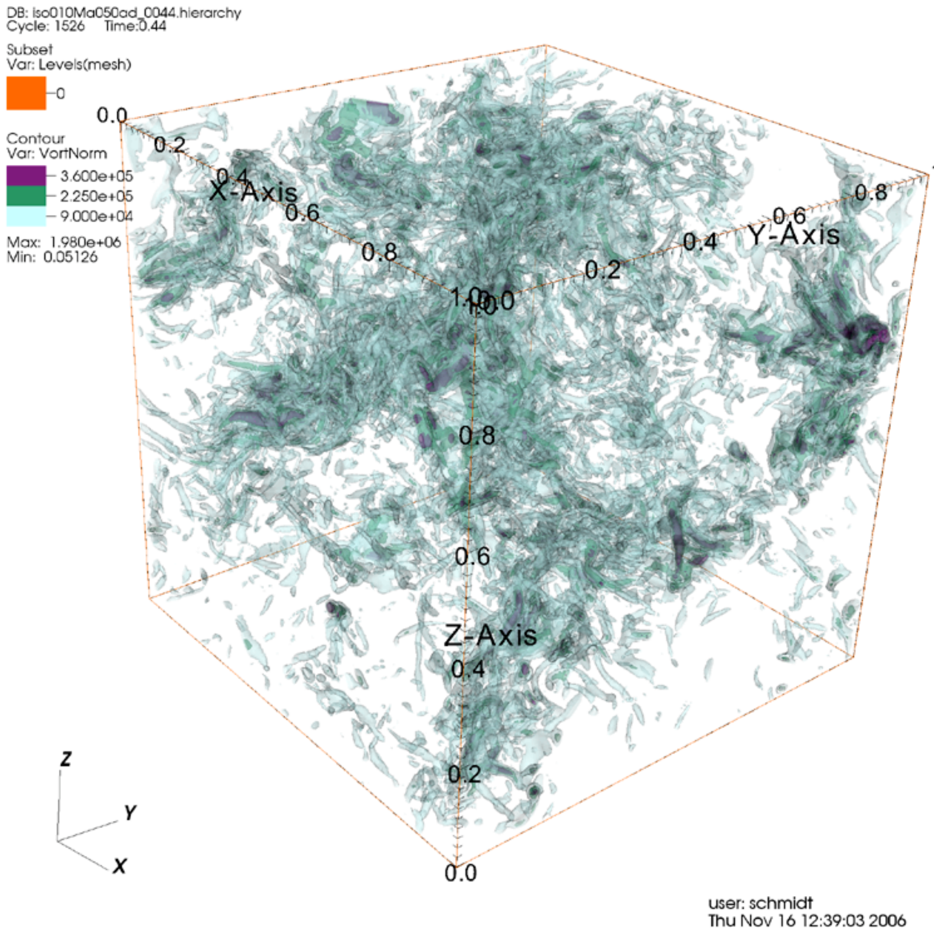


- Box with periodic boundary conditions
- Stochastic forcing at length scales ~ ½ box size
- Adiabatic or isothermal EOS
- Investigation of refinement criteria:
  - Characteristic Mach number $\mathrm{Ma} = 5$
  - Weight of solenoidal to compressive forcing modes $\zeta = 0.1$
  - Effective resolution $N_{\mathrm{eff}} = 192^3$, 1 refined level
- Production runs with $N_{\mathrm{eff}} = 768^3$, 1-2 refined levels

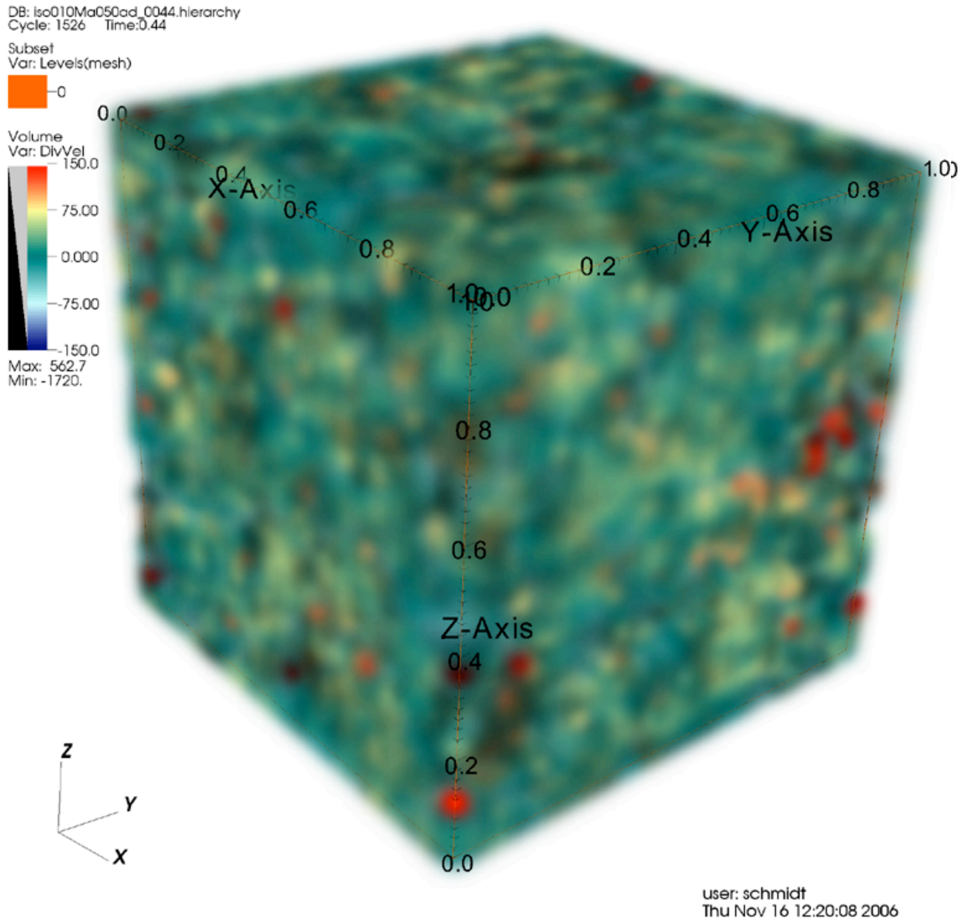
Static Grid Turbulence Simulation with Adiabatic EOS



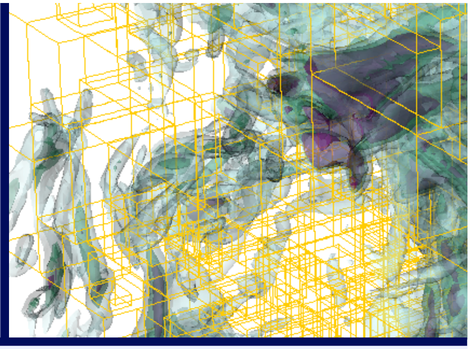
vorticity squared



divergence



Refinement by Gradients



- The conventional approach is to refine grids in the vicinity of steep gradients
- The gradient of the velocity field can be split into symmetric and antisymmetric parts:

$$v_{i,j} = S_{ij} + W_{ij}$$

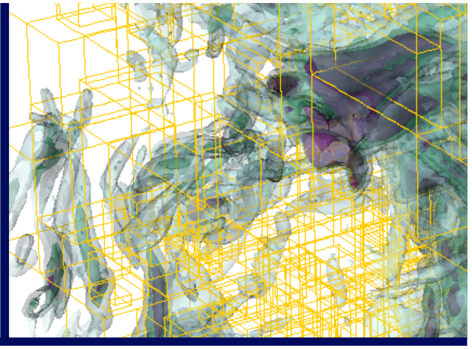
$$S_{ij} = \frac{1}{2}\left(v_{i,j} + v_{j,i}\right) \qquad W_{ij} = \frac{1}{2}\left(v_{i,j} - v_{j,i}\right)$$

$$|S| = \sqrt{2S_{ij}S_{ij}} \qquad |W| = \sqrt{2W_{ij}W_{ij}} = \omega$$

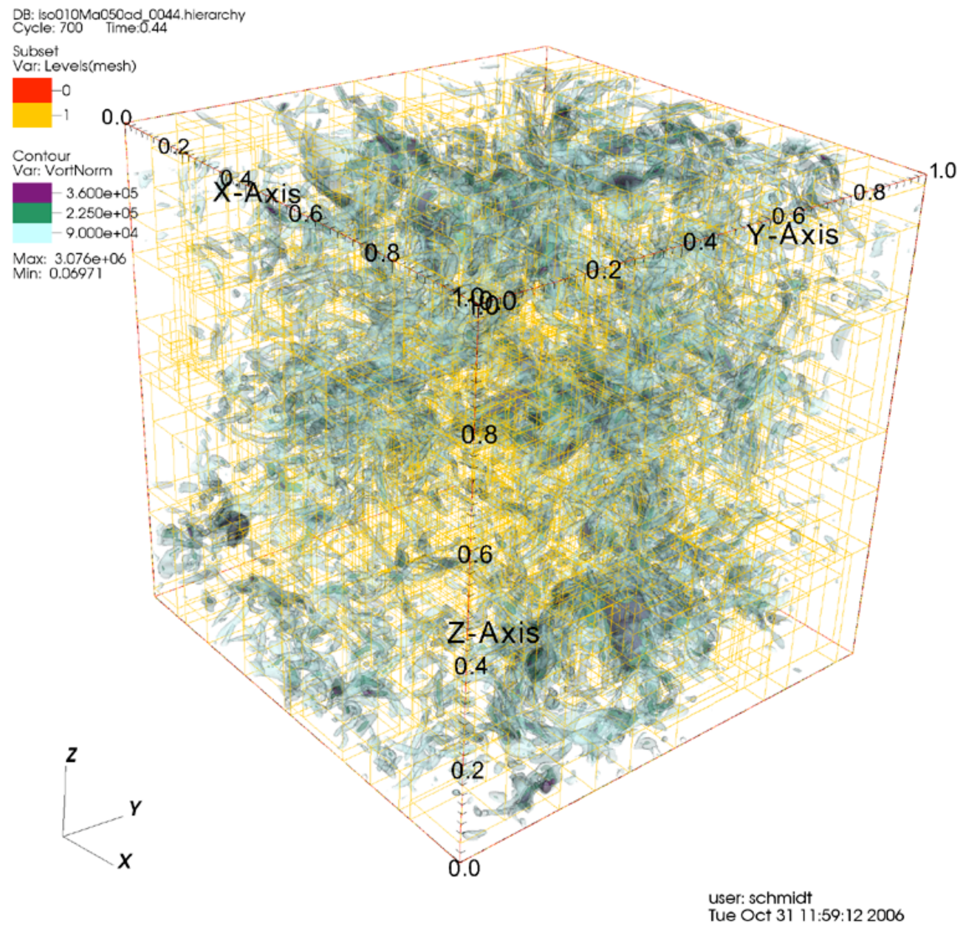
rate of strain — vorticity modulus

$$\omega \rightarrow |\boldsymbol{\nabla} \times \boldsymbol{v}|$$

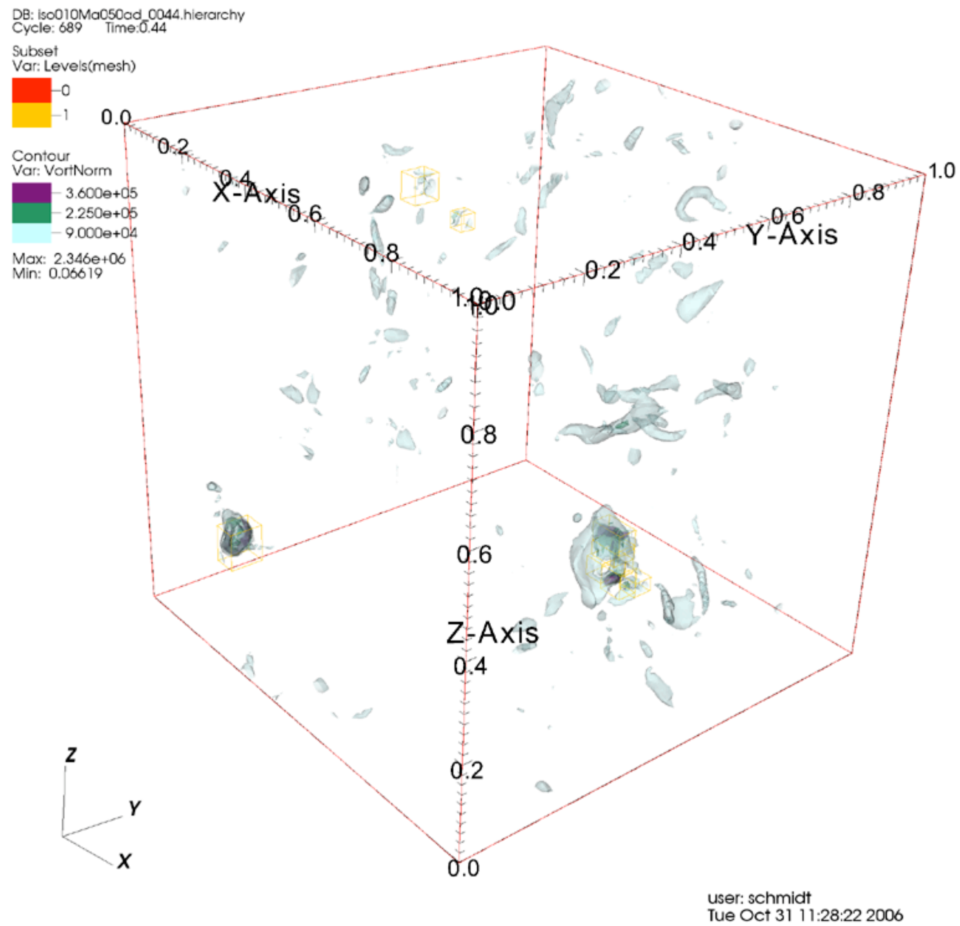
Refinement by global thresholds of ω^2 and $|S|^2$



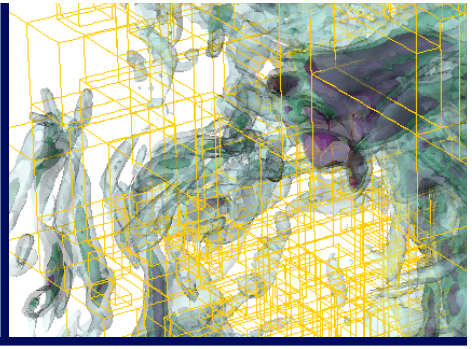
$\omega^2_{\text{thresh}} = |S|^2_{\text{thresh}} = 100$

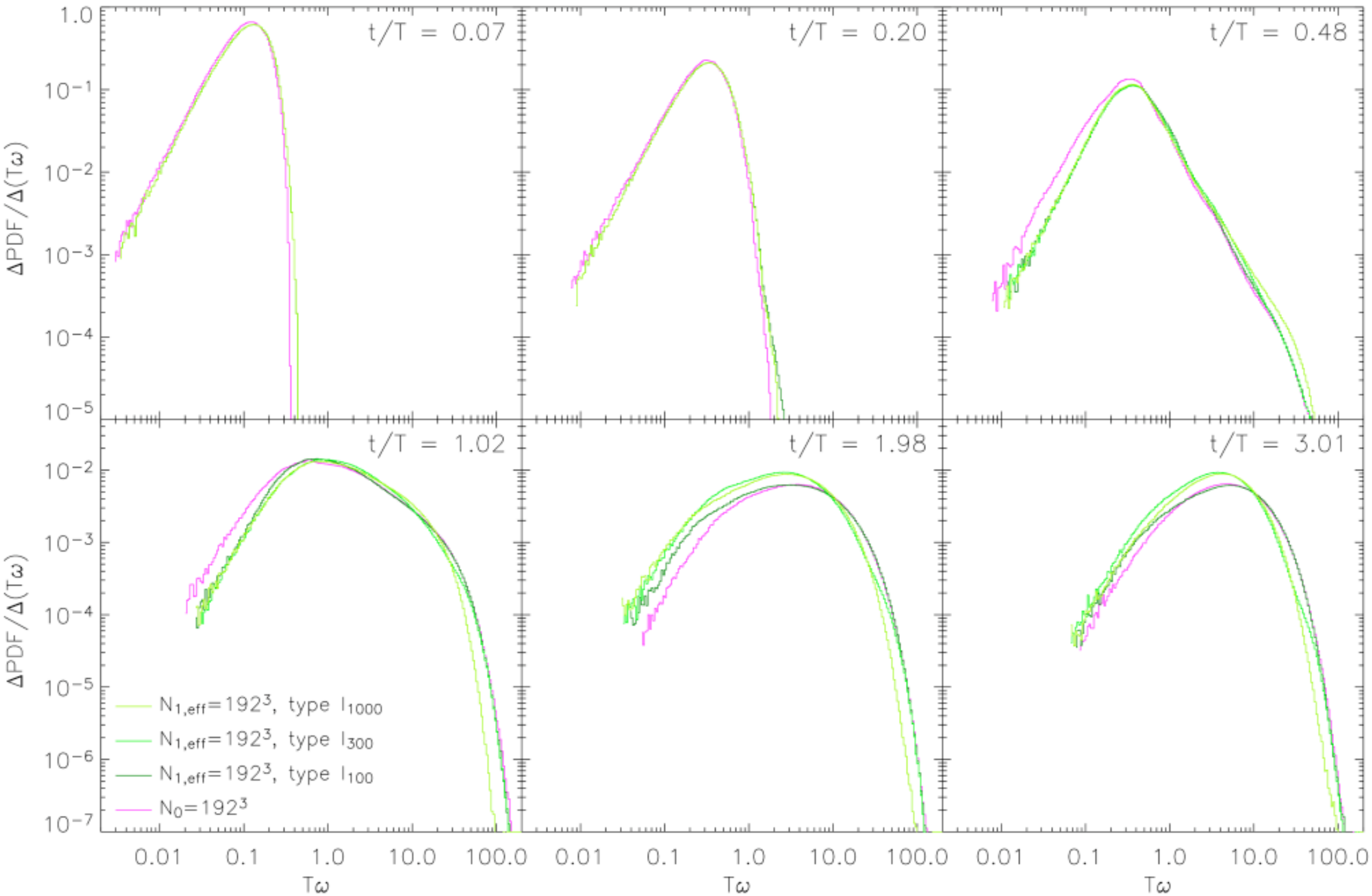


$\omega^2_{\text{thresh}} = |S|^2_{\text{thresh}} = 1000$



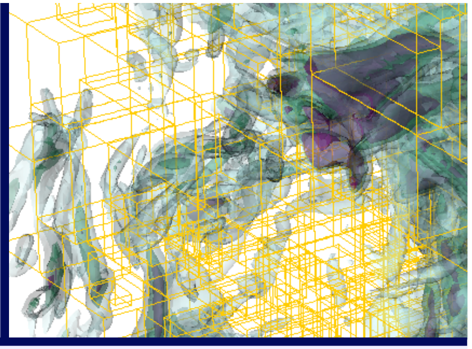
Refinement by global thresholds of ω^2 and $|S|^2$



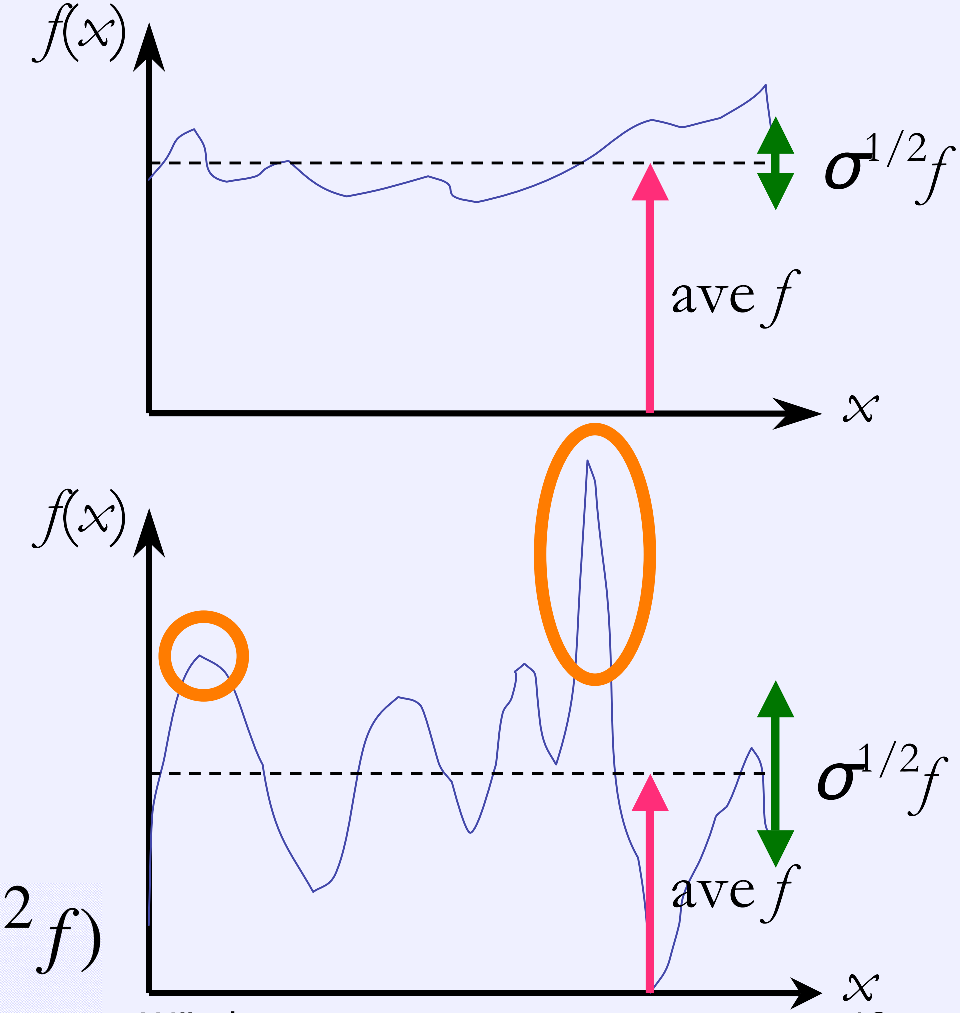


probability density function of vorticity (adiabatic EOS)

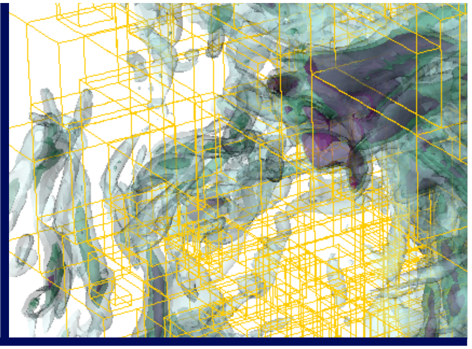
Thoughts on Averages and Deviations



- Global thresholds are mostly sensitive to the average
- But magnitude of fluctuations is given by the variance σ
- Small fluctuations should not trigger refinement
- Refine on i-th grid patch if $f(x) \geq C\lambda_i$ where $\lambda_i := \max_{G_i}(\mathrm{ave}\, f, \sigma^{1/2} f)$

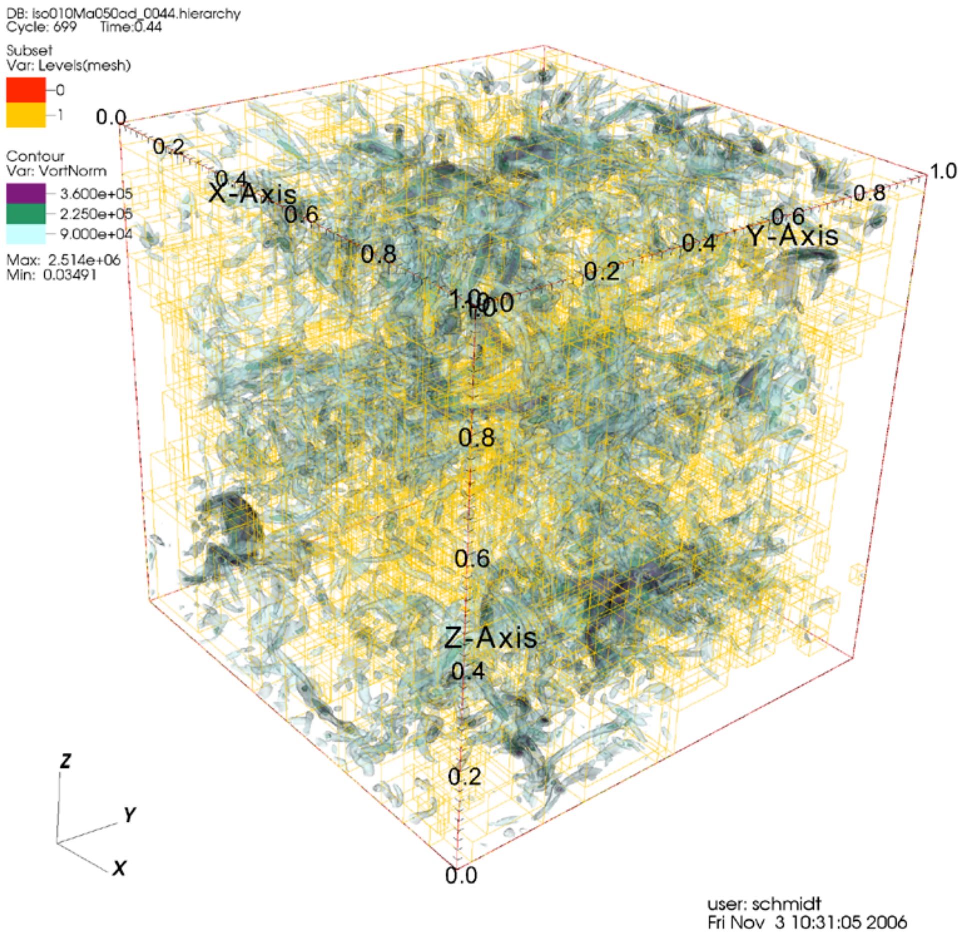


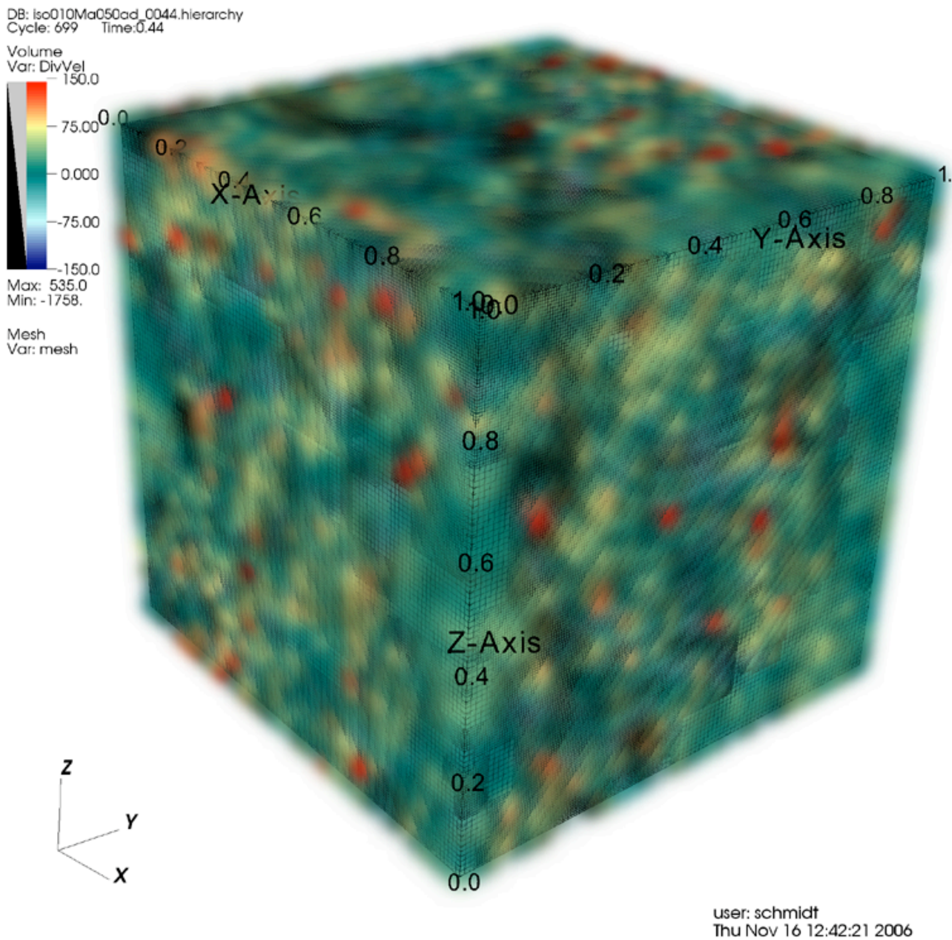
Refinement by regional variability of ω^2 and $|S|^2$



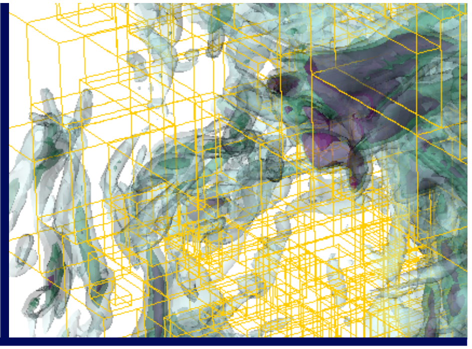
vorticity squared

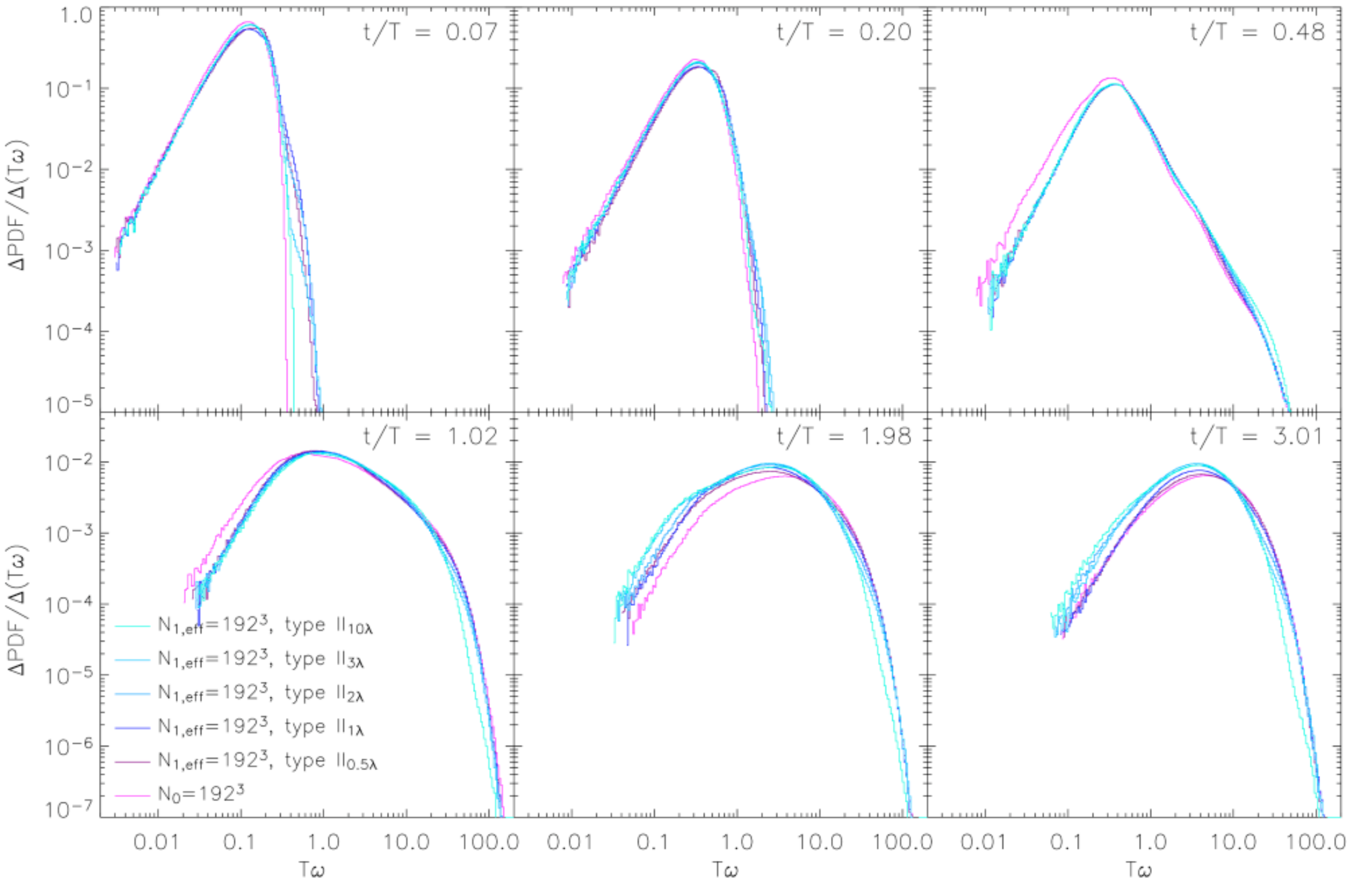
divergence





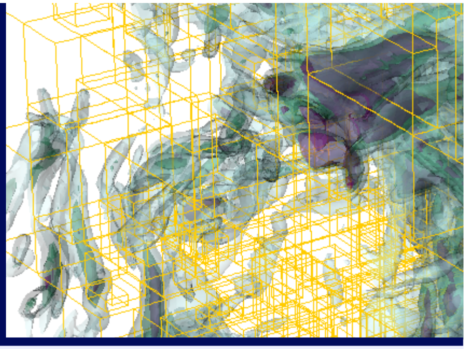
Refinement by regional variability of ω^2 and $|S|^2$





probability density function of vorticity (adiabatic EOS)

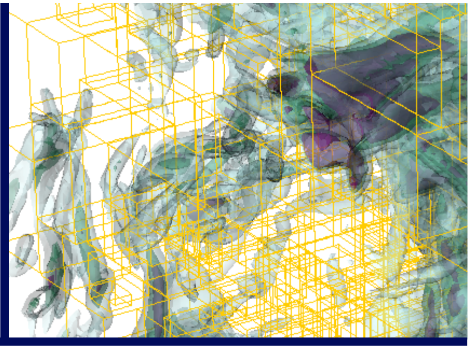
Towards AMR of Gravoturbulence



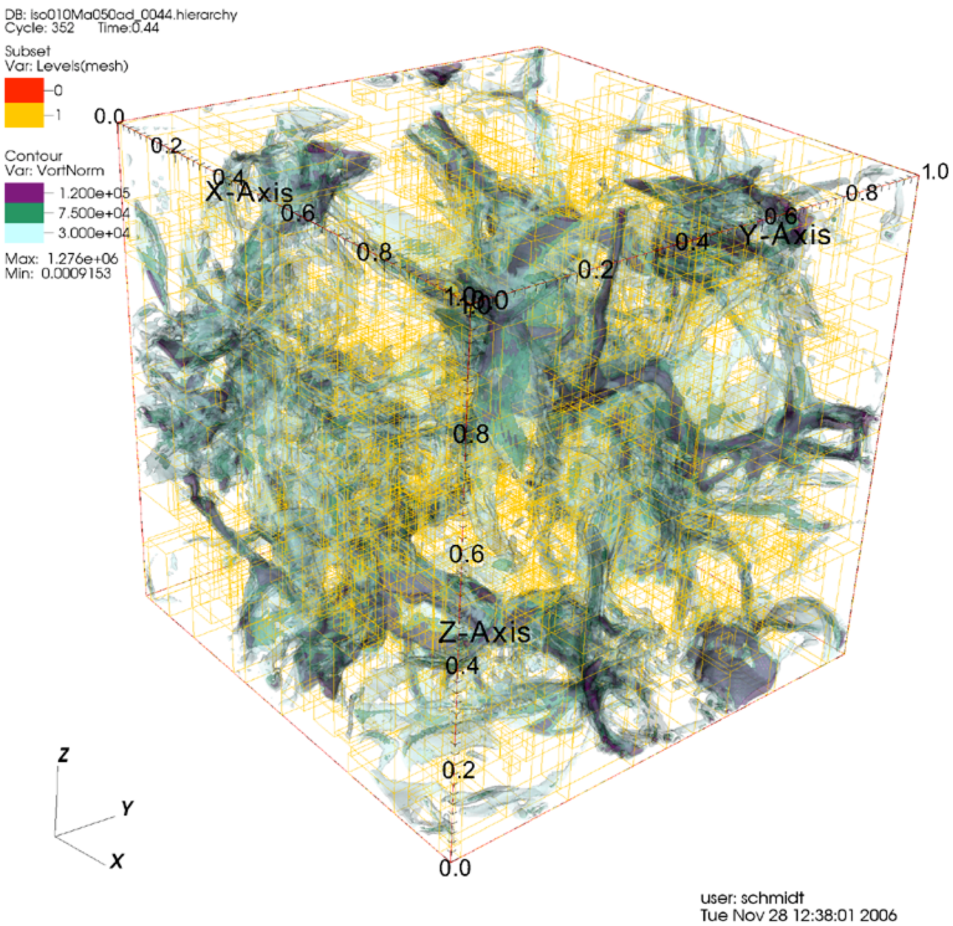
- If the Jeans length l_J becomes smaller then the box size, local gravitational collapse of compressed regions may ensue
- Additional refinement by l_J conceivable, but l_J is affected by turbulence (Bonazzola et al., 1987)
- Dynamical equation for rate of compression includes gravity term:

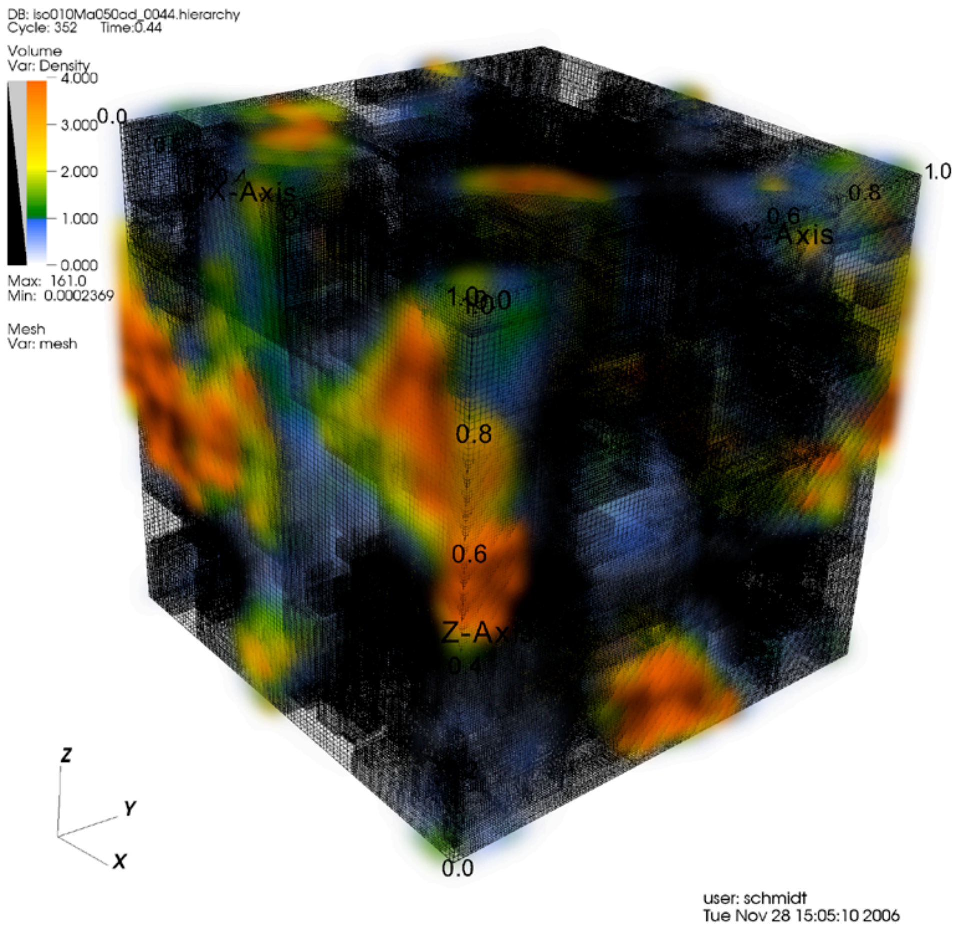
$$-\frac{\mathrm{D}}{\mathrm{D}t}d = \frac{1}{2}\left(|S|^2 - \omega^2\right) + \frac{1}{\gamma}c_s^2\nabla^2 \ln\rho + \boldsymbol{\nabla}\frac{1}{\gamma}c_s^2 \cdot \boldsymbol{\nabla}\ln\rho + \rho\nabla^2\frac{1}{\gamma}c_s^2 + 4\pi G\rho$$

Isothermal EOS with Refinement by Compression

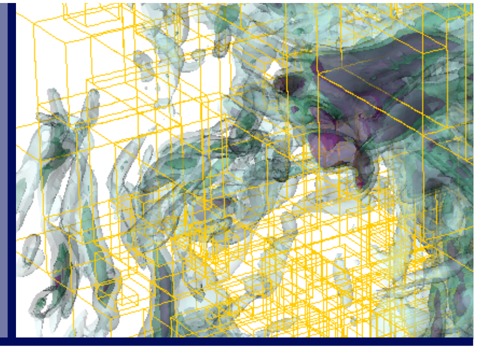


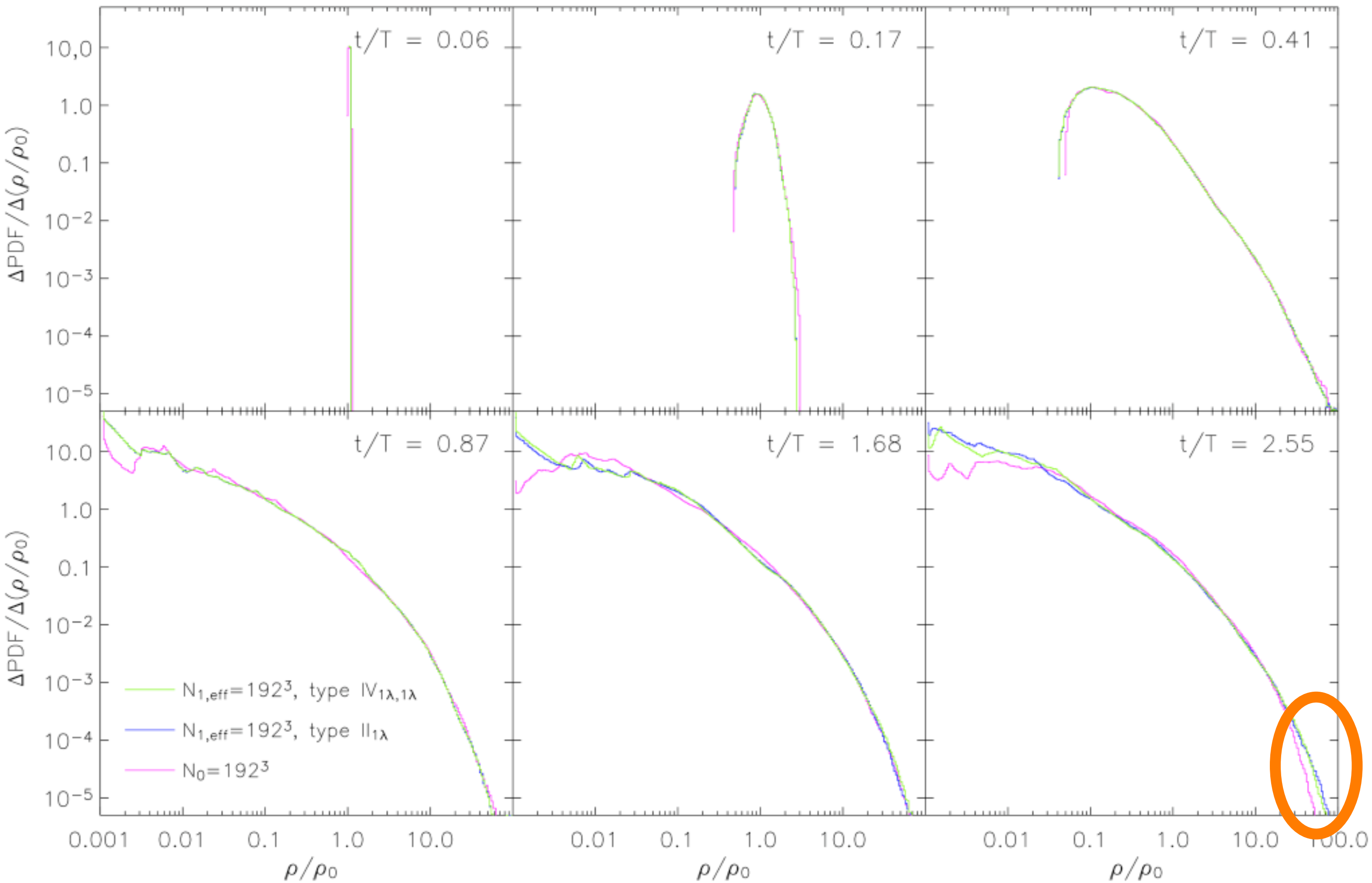
vorticity squared | mass density





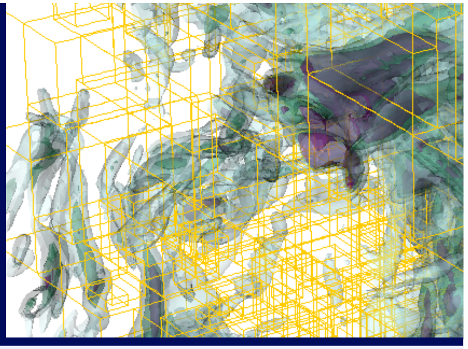
Refinement by ω^2 and $|S|^2$ vs. ω^2 and $-Dd/Dt$





probability density function of mass density (isothermal EOS)

SGS Turbulent Pressure



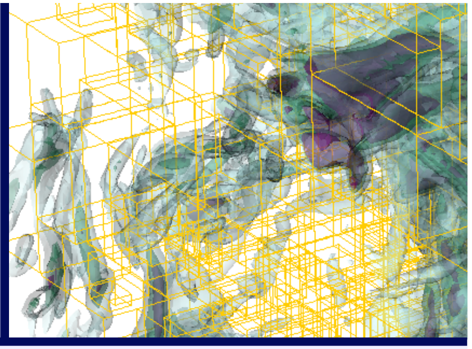
- Unresolved velocity fluctuations produce turbulent pressure
- In large eddy simulations, this pressure is given by the subgrid scale turbulence energy:

$$P_{\mathrm{sgs}} = \frac{2}{3}\rho k_{\mathrm{sgs}} = \frac{1}{3}\rho q_{\mathrm{sgs}}^2$$

- Turbulent pressure modifies the EOS:

$$P_{\mathrm{eff}} = P + P_{\mathrm{sgs}} = \rho\left(\frac{1}{\gamma}c_{\mathrm{s}}^2 + \frac{1}{3}q_{\mathrm{sgs}}^2\right)$$

SGS Turbulence Energy Model

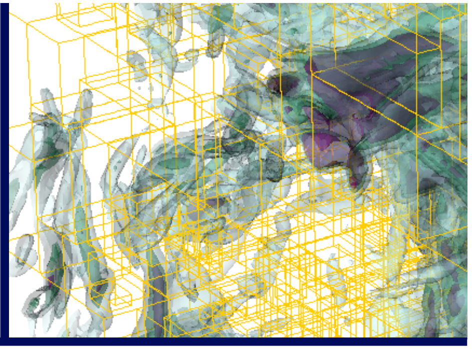


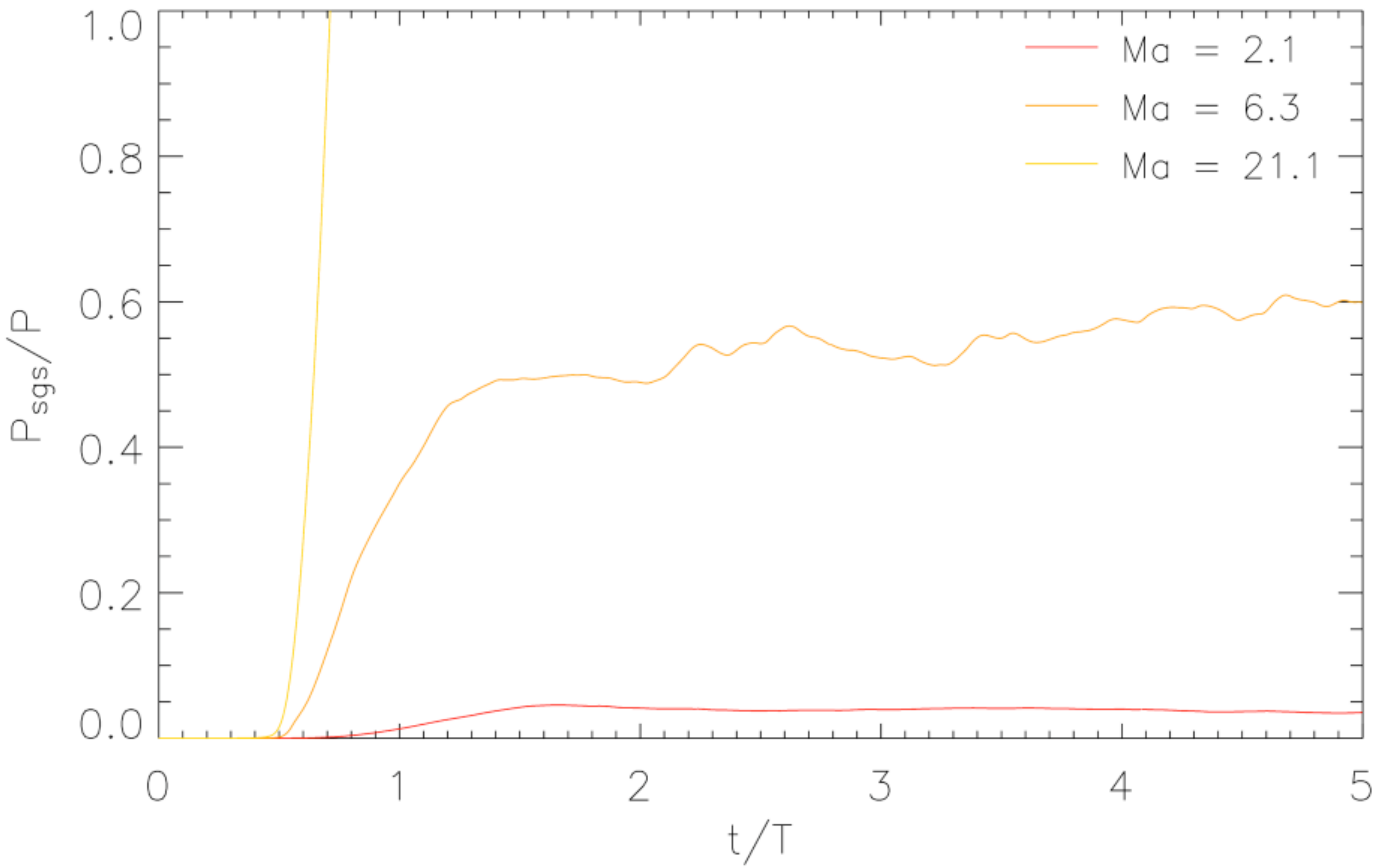
- Dynamical equation with lowest-order pressure-dilatation corrections adopted from the closures proposed by Sarkar (1992) for RANS:

$$
\begin{aligned}
\frac{\mathrm{D}}{\mathrm{D}t}k_{\mathrm{sgs}} - \frac{1}{\rho}\boldsymbol{\nabla}\cdot\left(\rho C_{\kappa}\Delta_{\mathrm{eff}}k_{\mathrm{sgs}}^{1/2}\boldsymbol{\nabla}k_{\mathrm{sgs}}\right)
&= \left(C_{\nu} - \alpha_2\frac{\sqrt{2k_{\mathrm{sgs}}}}{c_{\mathrm{s}}}\right)\Delta_{\mathrm{eff}}k_{\mathrm{sgs}}^{1/2}|S^*|^2 - \frac{2}{3}\left(1 - 8\alpha_4\frac{k_{\mathrm{sgs}}}{c_{\mathrm{s}}^2}\right)k_{\mathrm{sgs}}d \\
&\quad - \left(C_{\epsilon} - 2\alpha_3\frac{k_{\mathrm{sgs}}}{c_{\mathrm{s}}^2}\right)\frac{k_{\mathrm{sgs}}^{3/2}}{\Delta_{\mathrm{eff}}}.
\end{aligned}
$$

- Localised eddy-viscosity closure with test filtering (WS et al., 2005 & 2006)

LES of Supersonic Turbulence





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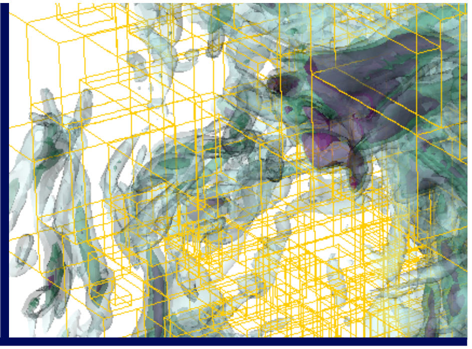
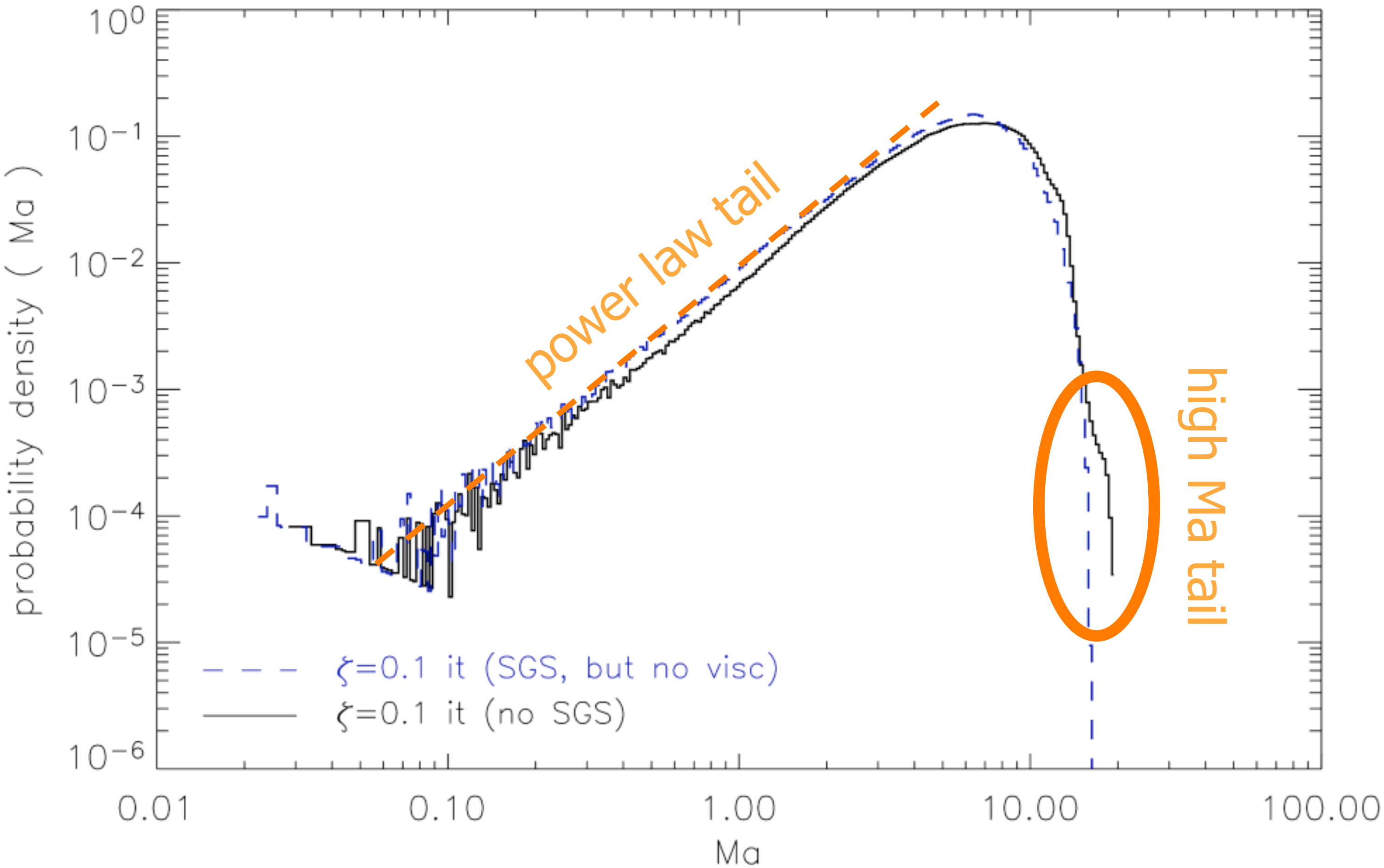
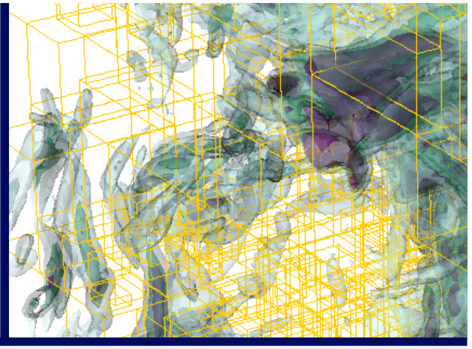
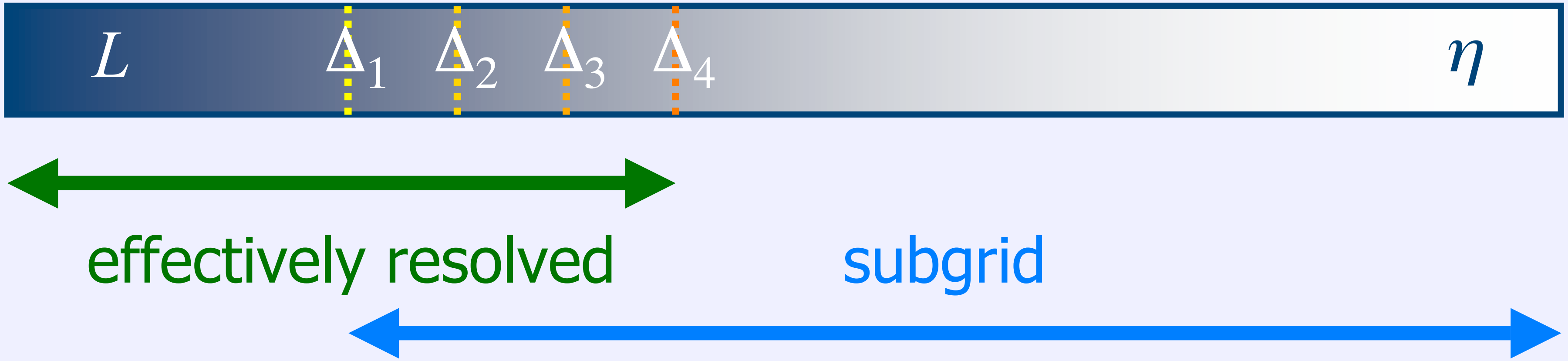




Outlook:
AMR + SGS = FEARLESS

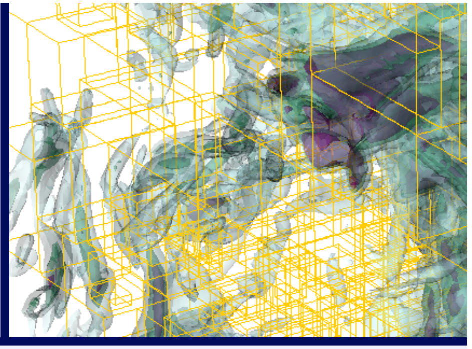




- Resolve shocks and collapsing regions with AMR
- SGS model treats asymptotically isotropic turbulence

Fluid m**E**chanics with **A**daptively
Refined **L**arge **E**ddy **S**imulation**S**

Astrophysical Applications



- Formation of the first stars (Abel et al., 2002)
- Galactic star formation in the turbulent interstellar medium (Mac Low & Klessen, 2004)
- Probabilistic model for the star formation rate in simulations of galaxy evolution à la Krumholz & McKee (2005)
- Intergalactic gas in clusters, etc.