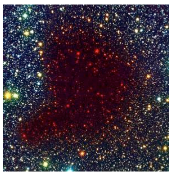
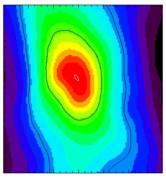
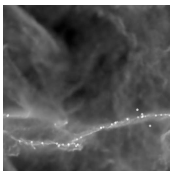
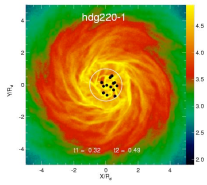
Sterne 2





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Literatur über Sterne

- Liste von empfohlenen Büchern

- Carroll, B.W., & Ostlie, D. A. 1996, "In Introduction into Moder Stellar Astrophysics" (Addison Welsey) -- Chapters 7 - 17
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- Allgemeine Bücher
  - Binney, J. & Merrifield, M. 1998, "Galactic Astronomy" (Princeton University Press)
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Wichtige theoretische Konzepte

- ***Sternaufbaugleichungen***
  (Hydrostatik in sphärischer Symmetrie)
- ***Virialsatz***
- ***Zustandsgleichungen im Sterninneren***
  (ideales Gas, strahlungsdominiertes Gas, entartetes Gas)
- ***Energieerzeugungsprozesse***
  (pp-Kette, CNO-Zyklus)

Sternaufbaugleichungen 1

$$\frac{dM_r}{dr} = 4\pi \rho r^2$$

Massenschalen

$$\frac{dP}{dr} = -\rho \frac{GM_r}{r^2}$$

Hydrostatisches Gleichgewicht

$\text{Gravitationskonstante} = G = 6,67 \times 10^{-8} \text{cm}^3 \text{g}^{-1} \text{s}^{-2}$

$$P = \frac{\rho}{m} kT$$

Gaszustandsgleichung

$\text{Boltzmann} - \text{Konstante} =$

$K = 1.38 \times 10^{-16} \text{erg K}^{-1}$

$$P = \frac{a}{3} T^4$$

Strahlungszustandsgleichung

$a = 7.56 \times 10^{-15} \text{ erg cm}^{-3} \text{ K}^{-4}$

Stefan-Boltzmann Konstante

Sternaufbaugleichungen 2

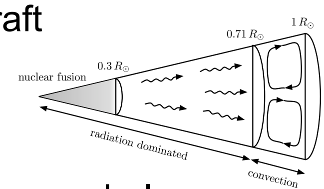
$$\frac{dP_{\text{rad}}}{dr} = -\frac{\kappa \rho}{c} F_{\text{rad}}$$

Druckgradient aus der Wechsel-wirkung mit dem Strahlungsfluss

$\kappa = \text{Opazität}$

$$F_{\text{rad}} = \frac{L_r}{4\pi r^2}$$

Zusammenhang zwischen Strahlungs-fluss und Leuchtkraft

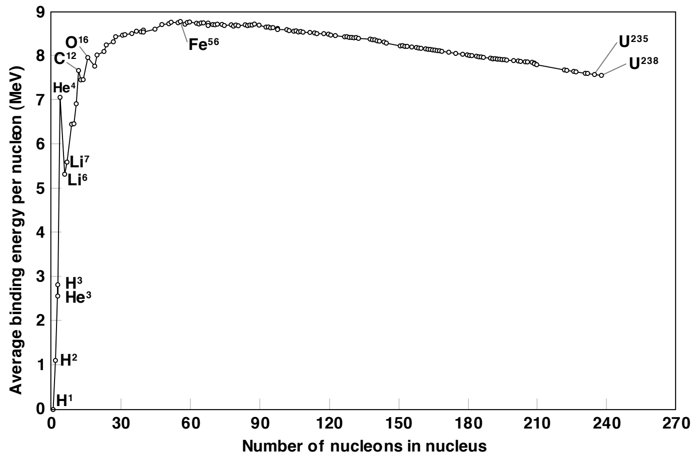


$$\frac{dL}{dr} = 4\pi r^2 \epsilon$$

Leuchtkraft in Massenschale

$\epsilon = \text{Energieerzeugungsrate pro Volumeneinheit}$

Energieproduktion in Sternen



Energieproduktion in Sternen

Unsere Sonne

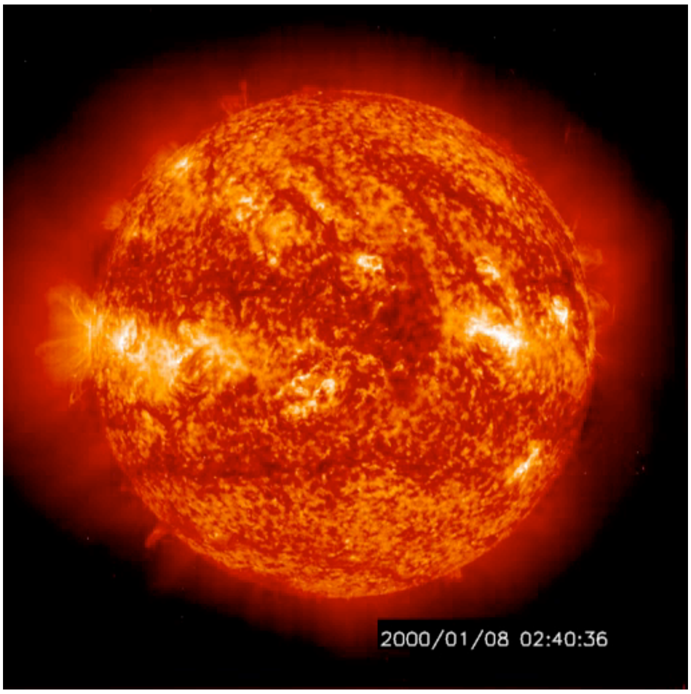
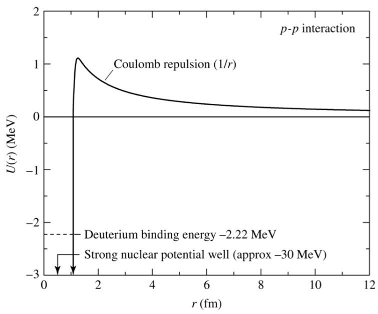


Bild: SOHO Satellit

Klassischer Ansatz



Carroll & Ostlie, Fig. 10.6

use Maxwell-Boltzmann velocity distribution.

peak:

$$\frac{1}{2}\mu_m\langle v^2\rangle = \frac{3}{2}kT = \frac{1}{4\pi\epsilon_0}\frac{Z_1Z_2e^2}{r}$$

μ_m = reduced mass

$$T = \frac{1}{6\pi\epsilon_0}\frac{Z_1Z_2e^2}{kr}$$

for $Z_1 = Z_2 = 1$ and $r \approx 1\,\mathrm{fm} = 10^{-15}\,\mathrm{m}$

$$\Rightarrow \quad T \approx 10^{10}\,\mathrm{K}$$

this is too hot compared to $T_c = 14 \cdot 10^6\,\mathrm{K}$ of the Sun. even in Maxwell-tail is considered (instead of peak)

Other process required for fusion ⇒ **tunneling**

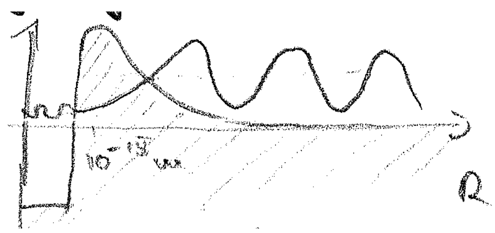
Tunneling

Approximation: tunneling becomes important when De Broglie wavelength of particle gets of order of the extent of potential well, i.e. of order fm.

kinetic energy:

$$\frac{1}{2}\mu_m\langle v^2\rangle = \frac{p^2}{2\mu_m} = \frac{(h/\lambda)^2}{2\mu_m} \qquad \text{(Heisenberg: } \lambda p_\lambda \approx h\text{)}$$

put together:



$$\frac{1}{4\pi\epsilon_0}\frac{Z_1Z_2e^2}{\lambda} = \frac{(h/\lambda)^2}{2\mu_m}$$

$$T = \frac{\mu_m}{12\pi\epsilon_0^2}\frac{Z_1^2Z_2^2e^4}{h^2k}$$

Again for collision of two protons: $\mu_m = \frac{1}{2}m_p$ and $Z_1 = Z_2 = 1$

$$\Rightarrow \quad T \approx 10^7\,\mathrm{K}$$

this temperature is consistent with estimates of stellar interior

Reaktionsrate

Putting all together:

$$R_{ix} = \left(\frac{2}{kT}\right)^{3/2} \frac{n_i n_x}{\sqrt{\mu_m \pi}} \int_0^\infty S(E) e^{-bE^{-1/2}} e^{-E/kT} \, dE$$

numbers:

$$R_{ix} = 6.48 \cdot 10^{-24} \frac{n_i n_x}{\mu_m Z_1 Z_2} S(E_0) \left(\frac{E_G}{4kT}\right)^{2/3} \exp\left(-3\left[\frac{E_G}{4kT}\right]^{1/3}\right) \frac{1}{m^3 s}$$

$$R_{ix} \propto n_i n_x \left(\frac{E_G}{4kT}\right)^{2/3} \exp\left(-3\left[\frac{E_G}{4kT}\right]^{1/3}\right)$$

$e^{-E/kT}$: high energy wing of MB distribution

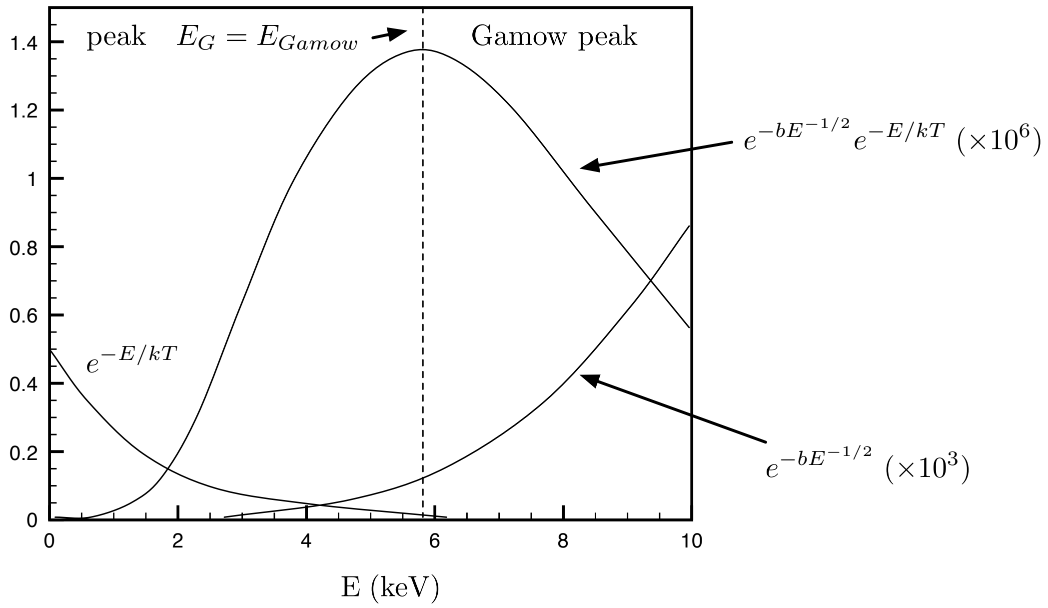
$e^{-bE^{-1/2}}$: comes from penetration probability

together **strongly peaked curve!**

Maximum at $E_0 = \left(\frac{bkT}{2}\right)^{2/3}$

besides this continuum description, there may be **Resonances** and **Electron screening** effects (polarization).

Reaktionsrate



Fusion von Wasserstoff

- Stars consists mostly of protons, so let´s starts with

  $$\mathrm{p} + \mathrm{p} \longrightarrow \mathrm{d} + \mathrm{e}^+ + \nu_\mathrm{e}$$

  this would be fast if it were not to need a **weak** interaction (to convert $\mathrm{p} \rightarrow \mathrm{n}$)

  $\mathrm{p} \rightarrow \mathrm{n} + \mathrm{e}^+ + \nu$ needs 1.8 MeV, but this is paid back by binding energy of deuteron, which is 2.22 MeV

  the p-p fusion rate is $5 \cdot 10^{13}\,\mathrm{m}^{-3}\mathrm{s}^{-1}$

  - a proton in center of Sun hangs around for $9 \cdot 10^9$ yr on average before it fuses with other proton
  - this sets **nuclear timescale**.

- once there is d, it quickly reacts with another p to build $^3\mathrm{He}$.

  $$\mathrm{p} + \mathrm{d} \longrightarrow {}^3\mathrm{He} + \gamma$$

Fusion von Wasserstoff

- now there are several ways to make $^4\mathrm{He}$.

  I) ${}^3_2\mathrm{He}$ fuses with ${}^3_2\mathrm{He}$ to make ${}^4_2\mathrm{He} + 2\mathrm{p}$

  $$ {}^3_2\mathrm{He} + {}^3_2\mathrm{He} \longrightarrow {}^4_2\mathrm{He} + 2\mathrm{p}$$

  $$\Delta E_\mathrm{eff} = 26.2\,\mathrm{MeV}$$

  II) or ${}^3_2\mathrm{He}$ fuses with ${}^4_2\mathrm{He}$ to ${}^7_4\mathrm{Be}$ and

  $$\begin{aligned} {}^3_2\mathrm{He} + {}^4_2\mathrm{He} &\longrightarrow {}^7_4\mathrm{Be} + \gamma \\ {}^7_4\mathrm{Be} + \mathrm{e}^- &\longrightarrow {}^7_3\mathrm{Li} + \nu_\mathrm{e} \\ {}^7_3\mathrm{Li} + \mathrm{p} &\longrightarrow 2\,{}^4_2\mathrm{He} \end{aligned}$$

  $$\Delta E_\mathrm{eff} = 25.2\,\mathrm{MeV}$$

  III) or ${}^3_2\mathrm{He}$ fuses again with ${}^4_2\mathrm{He}$ to ${}^7_4\mathrm{Be}$, but now

  $$\begin{aligned} {}^3_2\mathrm{He} + {}^4_2\mathrm{He} &\longrightarrow {}^7_4\mathrm{Be} + \gamma \\ {}^7_4\mathrm{Be} + \mathrm{p} &\longrightarrow {}^8_5\mathrm{B} + \gamma \\ {}^8_5\mathrm{B} &\longrightarrow {}^8_4\mathrm{Be} + \mathrm{e}^+ + \nu \\ {}^8_4\mathrm{Be} &\longrightarrow 2\,{}^4_2\mathrm{He} \end{aligned}$$

  $$\Delta E_\mathrm{eff} = 19.1\,\mathrm{MeV}$$

PP-Kette

- in summary: **pp Chain** (dominant for solar-type stars)

$${}^{1}_{1}\mathrm{H} + {}^{1}_{1}\mathrm{H} \longrightarrow {}^{2}_{1}\mathrm{H} + \mathrm{e}^{+} + \nu_{\mathrm{e}}$$
$${}^{2}_{1}\mathrm{H} + {}^{1}_{1}\mathrm{H} \longrightarrow {}^{3}_{2}\mathrm{H} + \gamma$$

different books give different branching ratios

69% [85%]

$${}^{3}_{2}\mathrm{He} + {}^{3}_{2}\mathrm{He} \longrightarrow {}^{4}_{2}\mathrm{He} + 2\,{}^{1}_{1}\mathrm{H}$$

ppI

31% [15%]

$${}^{3}_{2}\mathrm{He} + {}^{4}_{2}\mathrm{He} \longrightarrow {}^{7}_{4}\mathrm{Be} + \gamma$$

99.7%

$${}^{7}_{4}\mathrm{Be} + \mathrm{e}^{-} \longrightarrow {}^{7}_{3}\mathrm{Li} + \nu_{\mathrm{e}}$$
$${}^{7}_{3}\mathrm{Li} + {}^{1}_{1}\mathrm{H} \longrightarrow 2\,{}^{4}_{2}\mathrm{He}$$

ppII

0.3%

$${}^{7}_{4}\mathrm{Be} + {}^{1}_{1}\mathrm{H} \longrightarrow {}^{8}_{5}\mathrm{B} + \gamma$$
$${}^{8}_{5}\mathrm{Be} \longrightarrow {}^{8}_{4}\mathrm{Be} + \mathrm{e}^{+} + \nu_{\mathrm{e}}$$
$${}^{8}_{4}\mathrm{Be} \longrightarrow 2\,{}^{4}_{2}\mathrm{He}$$

ppIII

06.12.2012

PP-Kette

- in summary: **pp Chain** (dominant for solar-type stars)

$${}^{1}_{1}\mathrm{H} + {}^{1}_{1}\mathrm{H} \longrightarrow {}^{2}_{1}\mathrm{H} + \mathrm{e}^{+} + \nu_{\mathrm{e}}$$
$${}^{2}_{1}\mathrm{H} + {}^{1}_{1}\mathrm{H} \longrightarrow {}^{3}_{2}\mathrm{H} + \gamma$$

different books give different branching ratios

69% [85%]

$${}^{3}_{2}\mathrm{He} + {}^{3}_{2}\mathrm{He} \longrightarrow {}^{4}_{2}\mathrm{He} + 2\,{}^{1}_{1}\mathrm{H}$$

ppI

31% [15%]

$${}^{3}_{2}\mathrm{He} + {}^{4}_{2}\mathrm{He} \longrightarrow {}^{7}_{4}\mathrm{Be} + \gamma$$

99.7%

$${}^{7}_{4}\mathrm{Be} + \mathrm{e}^{-} \longrightarrow {}^{7}_{3}\mathrm{Li} + \nu_{\mathrm{e}}$$
$${}^{7}_{3}\mathrm{Li} + {}^{1}_{1}\mathrm{H} \longrightarrow 2\,{}^{4}_{2}\mathrm{He}$$

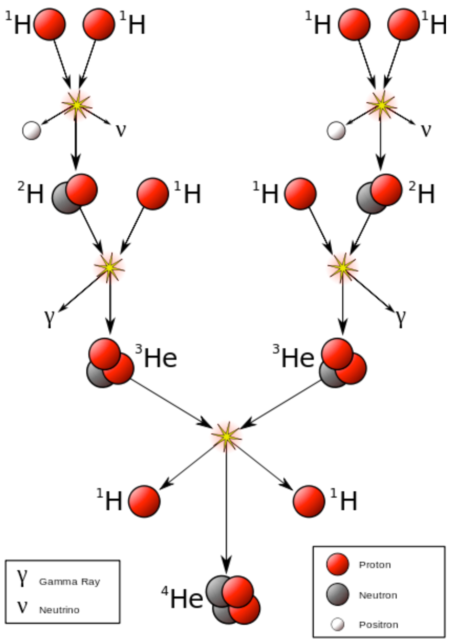
ppII

0.3%

$${}^{7}_{4}\mathrm{Be} + {}^{1}_{1}\mathrm{H} \longrightarrow {}^{8}_{5}\mathrm{B} + \gamma$$
$${}^{8}_{5}\mathrm{Be} \longrightarrow {}^{8}_{4}\mathrm{Be} + \mathrm{e}^{+} + \nu_{\mathrm{e}}$$
$${}^{8}_{4}\mathrm{Be} \longrightarrow 2\,{}^{4}_{2}\mathrm{He}$$

ppIII

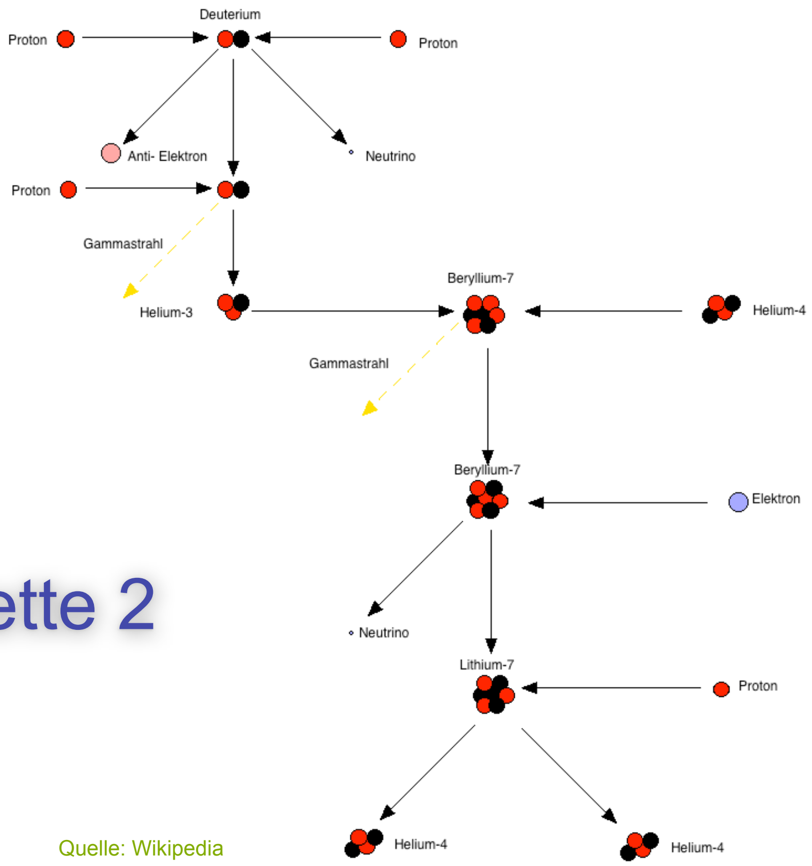
06.12.2012



PP-Kette 1

Quelle: Wikipedia

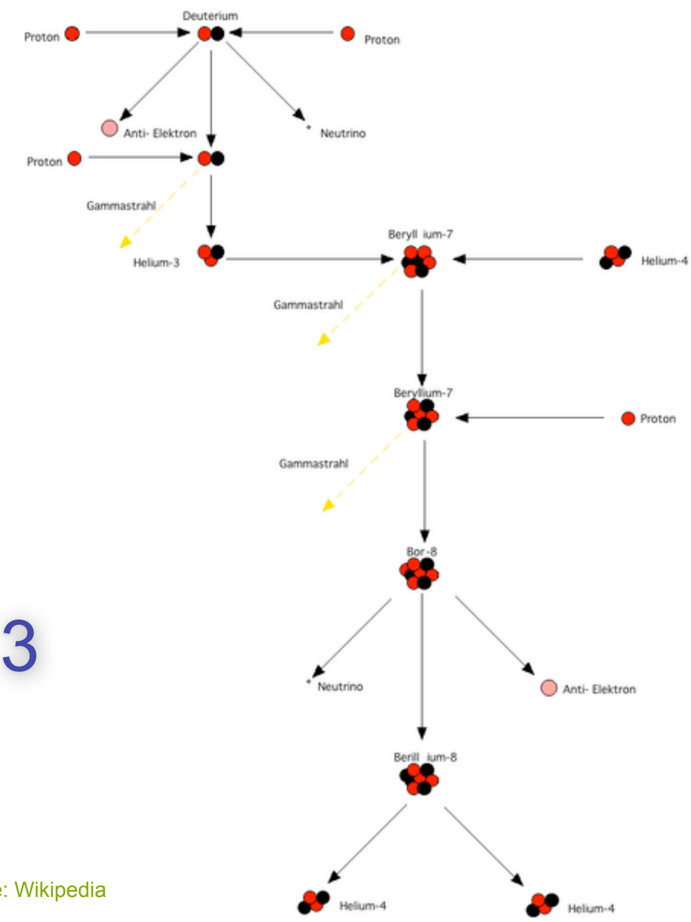
06.12.2012



PP-Kette 2

Quelle: Wikipedia

06.12.2012



PP-Kette 3

Quelle: Wikipedia

06.12.2012

CNO Zyklus

- **CNO cycle for massive star´s**
  - while the pp cycle can account for hydrogen burning in solar-type main sequence star´s it fails for massive star´s.
  - as $L \propto M^{5.5}$ (for stars similar to Sun) even a modest mass increase results in enormous luminosity gain. this is too much for the "modest" T^4-dependency of pp chain
    (recall: $T \propto M/R$)
    $\hookrightarrow$ something else must govern heat production in massive star´s.
  - steeper T-dependency required $\rightarrow$ larger Coulomb barrier $\rightarrow$ must involve heavy elements $\rightarrow$ but as their abundance (at best!) is very low, they must be recycled to prolong H burning
  - **CNO cycle with carbon, nitrogen, oxygen as catalysts!**

$$\mathrm{p} + {}^{12}_{6}\mathrm{C} \longrightarrow \underbrace{{}^{13}_{7}\mathrm{N}}_{{}^{13}_{6}\mathrm{C}+\mathrm{e}^{+}+\nu_e} + \gamma \qquad S(0) = 1.5\,\mathrm{keV\,barn}$$

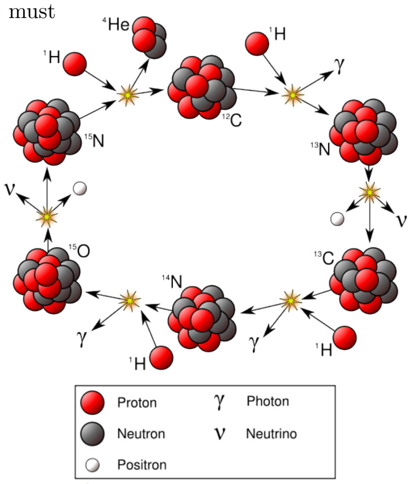
$$\mathrm{p} + {}^{13}_{6}\mathrm{C} \longrightarrow {}^{14}_{7}\mathrm{N} + \gamma \qquad S(0) = 5.5\,\mathrm{keV\,barn}$$

$$\mathrm{p} + {}^{14}_{7}\mathrm{N} \longrightarrow \underbrace{{}^{15}_{8}\mathrm{O}}_{{}^{15}_{7}\mathrm{N}+\mathrm{e}^{+}+\nu_e} + \gamma \qquad S(0) = 3.3\,\mathrm{keV\,barn}$$

$$\mathrm{p} + {}^{15}_{7}\mathrm{N} \longrightarrow {}^{12}_{6}\mathrm{C} + {}^{4}_{2}\mathrm{He} \qquad S(0) = 78\,\mathrm{keV\,barn}$$

the net result is

$$4\mathrm{p} \longrightarrow {}^{4}_{2}\mathrm{He} + 3\gamma + 2\mathrm{e}^{+} + 2\mathrm{e}\nu \qquad \Delta E_{\mathrm{eff}} = 23.8\,\mathrm{MeV}$$



Quelle: Wikipedia

06.12.2012

CNO Zyklus

– just like in pp, there are several CNO chains

$$^{12}_{6}\mathrm{C} + ^{1}_{1}\mathrm{H} \longrightarrow ^{13}_{7}\mathrm{N} + \gamma$$
$$^{13}_{7}\mathrm{N} \longrightarrow ^{13}_{6}\mathrm{C} + \mathrm{e}^{+} + \nu_{\mathrm{e}}$$
$$^{13}_{6}\mathrm{C} + ^{1}_{1}\mathrm{H} \longrightarrow ^{14}_{7}\mathrm{N} + \gamma$$
$$^{14}_{7}\mathrm{N} + ^{1}_{1}\mathrm{H} \longrightarrow ^{15}_{8}\mathrm{O} + \gamma$$
$$^{15}_{8}\mathrm{O} \longrightarrow ^{15}_{7}\mathrm{N} + \mathrm{e}^{+} + \nu$$

99.96%:

$$^{15}_{7}\mathrm{N} + ^{1}_{1} \longrightarrow ^{12}_{6}\mathrm{C} + ^{4}_{2}\mathrm{He}$$

0.04%:

$$^{15}_{7}\mathrm{N} + ^{1}_{1}\mathrm{H} \longrightarrow ^{16}_{8}\mathrm{O} + \gamma$$
$$^{16}_{8}\mathrm{O} + ^{1}_{1}\mathrm{H} \longrightarrow ^{17}_{8}\mathrm{F} + \gamma$$
$$^{17}_{8}\mathrm{F} \longrightarrow ^{17}_{8}\mathrm{O} + \mathrm{e}^{+} + \nu_{\mathrm{e}}$$
$$^{17}_{8}\mathrm{O} + ^{1}_{1}\mathrm{H} \longrightarrow ^{14}_{7}\mathrm{N} + ^{4}_{2}\mathrm{He}$$

– $\epsilon_{CNO} = 8.24 \cdot 10^{-24} \dfrac{\mathrm{erg/s}}{\mathrm{cm^3\, g^2}}\, \varrho X Z T_6^{19.9}$ at $T_6 \approx 15$

very steep T dependency:

$$\epsilon_{\mathrm{CNO}} \propto T^{20} \quad \text{at} \quad T_6 \approx 15$$

Vergleich PP und CNO

– for Sun ($T_6 = 15,\ X = 0.7,\ Z \approx 0.02$)

$$\epsilon_{\mathrm{pp}} \approx 10 \cdot \epsilon_{\mathrm{CNO}}$$

only 10% of Sun´s energy production from CNO [Note: Phillips (**) gives 2%]

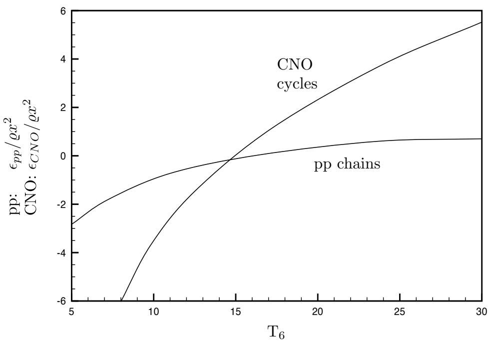
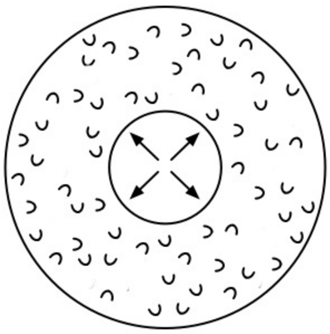
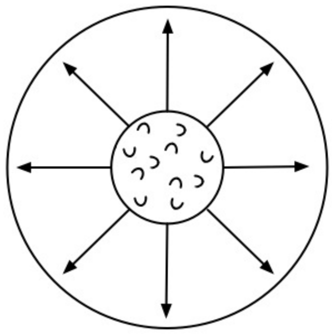


Figure 9.4:

– pp chains are lower in efficiency and energy yield than CNO (once CNO becomes possible)

$$\left.\begin{array}{r}\epsilon_{\mathrm{pp}} \approx 11 \ldots 17\,\mathrm{MeV} \\ \epsilon_{\mathrm{CNO}} \approx 23 \ldots 27\,\mathrm{MeV}\end{array}\right\} \text{for solar models}$$

Vergleich im Sternaufbau

lower main sequence	upper main sequence
$M < 1.5\,\mathrm{M}_\odot$ (M to F0)	$M > 1.5\,\mathrm{M}_\odot$ (F0 to O)
pp-chain	CNO-cycle
$\epsilon_{\mathrm{pp}} \propto T^4$	$\epsilon_{\mathrm{CNO}} \propto T^{20}$
low T-dependency; less concentrated energy source → small T-gradients	in center larger flux and steep temperature gradient
Radiative Core	**Convective Core**
T_c >	$20 \cdot 10^6$ > T_c
surface H neutral ; then ionization and rapid increase of opacity → steep T gradient.	surface hot and ionized modest temperature gradient.
convective envelope	**radiative envelope**
	
the smaller M, the further convective zone reaches into star.	the larger m, the larger the convective core