

Exercises for Introduction to Cosmology (WS2011/12)

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Exercise sheet 2

1. Perturbations on a hypothetical static universe

In the lecture we have shown that a cosmological constant can be regarded as an outward force that scales linearly with distance: $\vec{f}_\Lambda = (\Lambda/3)\vec{r}$. We saw that a static universe could be created in this way, if one fine-tunes Λ . We saw that this solution is unstable.

- (a) If the density of this universe is equal to what we typically believe the current Universe has ($\rho \simeq 2.3 \times 10^{-30}$ gram/cm³), then what would be the e-folding growth time scale of this instability?
- (b) If we would make a universe in which Λ is *exactly* fine-tuned to have a static universe, then a global instability would not happen. But if we impose at some time t_0 some density perturbations of order 1% (i.e. $\delta\rho/\langle\rho\rangle \simeq 1\%$) with a typical spatial length scale λ , what will happen (qualitatively)?
- (c) Estimate how long will it take for this evolution to become non-linear. Does this depend on λ ? Interpret what this non-linear evolution would mean (i.e. what would an observer in this universe see)?

2. Light propagation in an exponentially expanding Universe

Suppose we have a Universe in which the metric is given by the following line element:

$$ds^2 = -c^2 dt^2 + e^{2At}(dx^2 + dy^2 + dz^2) \quad (7)$$

with $A = \text{constant}$.

- (a) Is the metric spatially flat?
- (b) Is the metric flat in spacetime?
- (c) Does this Universe have a Big Bang?
- (d) What would the the Hubble constant for this case?
- (e) Suppose a light signal is sent from point $(x, y, z) = (0, 0, 0)$ in x -direction at time $t = 0$. Give its position as a function of time: $x(t)$.

3. Our own exponentially expanding Universe

Since about 5 Gyrs the real Universe is in a state of exponential expansion as a result of Dark Energy, i.e. a cosmological constant.

- (a) Does this mean that in the very far future even the Earth will be ripped apart?
- (b) Could it mean that in the (somewhat less far) future the orbit of the Moon and the Earth is widened because of the “creation of space” between the Earth and the Moon due to the expansion of the Universe?