Exercises for Introduction to Cosmology (WS2011/12)

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Exercise sheet 3

1. **The metric of Robertson-Walker space**

   (a) Show that the spatial line element of the Friedmann metric,

   $$ds^2 = dr^2 + f_K^2(r) d\omega^2 \tag{8}$$

   can equivalently be written in the form

   $$ds^2 = \frac{d\bar{r}^2}{1 - K\bar{r}^2} + \bar{r}^2 d\omega^2 \tag{9}$$

   (b) For both forms of the metric, calculate the surface area of a sphere with constant unit radius $r = 1$ resp. $\bar{r} = 1$.

2. **Conformal time in an empty flat universe with Λ**

   (a) As we saw in the lecture, for an empty flat space with cosmological constant Λ the scale factor goes as $a(t) = Ae^{Ht}$. Show that $H = \sqrt{\Lambda/3}$.

   (b) Sometimes the coordinate time t in the Friedmann metric is replaced by the *conformal time* η, defined by

   $$ds^2 = a^2 \left[-c^2 d\eta^2 + d^2 r + f_K^2(r) d\omega^2 \right] \tag{10}$$

   Find η as a function of t for an empty flat universe with cosmological constant Λ.

3. **Why is the Universe flat?**
   It seems that to the best of current measurements the Universe is flat. Scientists have been initially really puzzled by this (before they came up with inflation to explain it). To see why this is so strange let us follow for simplicity the evolution of a Universe with $\Omega_{\Lambda,0} = 0$, $\Omega_{m,0} = 0$ and $\Omega_{r,0} = 0.99$, i.e. the Universe is filled entirely with radiation and is, at present, just 1% off from the critical density.

   (a) How close (percent-wise) to the critical density was the density of the Universe at $z = 10^4$; and at $z = 10^8$? *Hint:* First order estimate is fine.

   (b) Explain, on the basis of this result, why this makes the fact that the Universe today has $|\Omega_{K,0}| \lesssim 10^{-2}$ such an apparent "coincidence".