Continuity Equation

Problem sheet 5

26/05/2009

Flux-Limiter

Aim of this exercise sheet is the improvment of the numerical Doner-Cell scheme for the solution of the continuity equation via high-resolution Flux-Limiters.

1. Extend in your advection-code the calculation of the flux over the interfaces by a second-order corrector term:

$$\begin{aligned} F_{i-1/2} &= \rho_{i-1}\max(v_{i-1/2},0) \\ &+\rho_i \min(v_{i-1/2},0) \\ &+\frac{1}{2}|v_{i-1/2}|\left(1-\frac{\Delta t}{\Delta x}|v_{i-1/2}|\right)\delta^n_{i-1/2} \end{aligned}$$

   With $\delta^n_{i-1/2} = \rho^n_i - \rho^n_{i-1}$ this would give the Lax-Wendroff method.

2. We want to change this Lax-Wendroff flux by multiplying an arbitrary flux-limiter function $\Phi(\theta)$:

$$\delta^n_{i-1/2} = \Phi(\theta)_{i-1/2}(\rho^n_i - \rho^n_{i-1})$$

   which depends on the "smoothness" θ of the advected data in the upwind direction:

$$\theta_{i-1/2} = \frac{(\rho^n_{i-1}-\rho^n_{i-2})\max(v_{i-1/2},0) + (\rho^n_{i+1}-\rho^n_i)\min(v_{i-1/2},0)}{(\rho^n_i - \rho^n_{i-1})v_{i-1/2}}$$

3. Implement the following flux-limiters $\Phi(\theta)$:

Method	$\Phi(\theta)$
Linear Methods:	
Upwind	0
Lax-Wendroff	1
Beam-Warming	θ
Fromm	$\frac{1}{2}(1+\theta)$
High resolution limiters:	
van Leer	$\frac{\theta+\|\theta\|}{1+\|\theta\|}$
Monoticed Center difference (MC)	$\max(0,\min(2,\frac{1}{2}(1+\theta),2\theta))$
Superbee	$\max(0,\min(1,2\theta),\min(2,\theta))$