# Isothermal Hydrodynamics

Problem sheet 6

2/06/2009

## Isothermal Shock Tube

Now we learned, how to advect a function over a grid. So we nearly gathered all the knowledge to build up the hydro code. Aim of this exercise sheet is the expansion from the 1D-advection code to a full (isothermal) 1D-Hydro-Solver.

1. Write down the 1D equations for isothermal hydrodynamics.

2. Expand your code: Calculate in each timestep the isothermal pressure distribution. Use isothermal sound speed $c_s = 1$.

3. If we want to consider the time evolution of the velocity distribution, the corresponding timestep (Courant-Friedrichs-Lewy condition, CFL) changes with time, too.

   Expand your code: Calculate at the beginning of each iteration the new possible timestep. Use a CFL factor of 0.5 and consider also the signal velocity $c_s$.

4. Solve the continuity equation numerically with flux limiters learned from problem sheet #5.

5. To solve the momentum equation numerically, we'll split the equation into two steps:

   $$\begin{aligned} \partial_t(\rho v) &= -\partial_x[(\rho v)v] \text{ (transport step)} \\ \partial_t(\rho v) &= -\partial_x P \text{ (source step)} \end{aligned}$$

   The first one describes the advection of the density flux $\rho v$ and is an exact copy of the continuity equation. The second one describes the additionally acting hydrodynamic pressure force.

   Expand your code:

(a) Apply the flux limiter learned from problem sheet #5 when transporting the momentum.

(b) Solve the pressure force equation after the above described velocity transport during a second velocity update.

6. Test your final code with the isothermal shock tube problem:
100 physical grid cells (+boundaries), $\Delta x = 1$, $c_s(x,t) = 1$, closed boundaries.

$$\begin{aligned} \rho(x, t=0) &= 2\Theta(50 - x) + 1 \\ v(x, t=0) &= 0, \end{aligned}$$

where

$$\Theta(x) = \begin{cases} 0 & x \leq 0 \\ 1 & x > 0 \end{cases}.$$

Integrate up to $t = 30$.