Lecture Numerical Fluid Dynamics:

Lecture given by

C.P. Dullemond and V. Springel

Script has gained from input by
H.H. Wang and S. Ataiee

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About the lecture

Introduction

The topic of *hydrodynamics* or *fluid dynamics* describes the time-dependent or stationary flow of fluids or gases. The name originates from fluid flow (hence the names 'hydro' and 'fluid'), but in physics this name is also used to describe *gas* flows, and in astrophysics it is nearly exclusively used in that way.

Hydrodynamics is a topic that is of fundamental importance to many applications in industry. For instance, it is used to model the flow around airfoils and entire airplanes, the aerodynamics of sports cars, how water flows through pipes and ducts, the aerodynamic stability of constructions, complex combustion processes in chemical reactors, etc. It also plays an important role in many scientific studies, for instance in meteorology and climate research (modeling ocean and air currents and the formation of weather fronts) and in astrophysics where it is used for practically all topics: star- and planet formation, the formation of galaxies, the formation of structre in the early universe, the accretion of gases onto black holes and neutron stars, the colliding winds of evolved stars etc.

In practically all applications the problems are already so complex that analytical methods are inadequate. The solution is almost always sought using *numerical methods*, which, implemented in computer programs, can give numerical solutions for given intial- and or boundary conditions.

This lecture aims to give an introduction to these numerical methods. We will focus on:

- Gaining an understanding of the mathematical properties of the equations of hydrodynamics, which underly many of the algorithms to solve them.
- Gaining an understanding of the methods, how and why they work and what their limitations are.
- Hands-on experiments (in the practicum) by self-made programs for simple problems
- Hands-on experiments with some off-the-shelf programs for more complex and realistic problems
- Example applications of the methods, and their background.

Various methods

There is no single numerical recipe for all hydrodynamic problems. For instance, for modeling weather patterns on Earth one requires methods than can handle the strong contrast of distance scales in the atmosphere, where the vertical height (roughly 20 km) is much less than the size of a typical depression (1000 km). The problem of flow around airfoils (=wings) typically require methods that have their grids adapted to the wing-shape. In astronomy one typically has 3-D flows of interstellar gas which can be extremely supersonic (involving very strong shocks) and can have extreme density contrasts (factors of a million) and spatial scales differing by similar factors. In other applications one may have more interest in detailed steady-state flows which are best solved using special-purpose potential solvers. In some cases a detailed treatment of turbulence is required, which again puts different constraints on the numerical methods used.

Also, each problem may involve different additional physics. For instance, astrophysical hydrodynamics usually involves the interaction between gas and a radiation field. This requires the combined hydrodynamic and radiative transfer equations to be solved. This is called *radiation-hydrodynamics*, and is so complex that in many applications it is still in its infancy. Also in astrophysics magnetic fields play a crucial role: gas motions can generate (or at least amplify) magnetic fields through dynamo action and these fields can strongly affect the motion of the gas. An example of this is the magnetosphere of the sun, where solar eruptions are clearly an interplay between gas and magnetic fields. This is called *magneto-hydrodynamics*, and is a field of applied mathematics that is still under development. In meteorology, in addition to the interaction with radiation from the sun, the air currents are affected by the evaporation and condensation of water, and the precipitation that follows from this. Moreover, in this case the coriolis forces by the Earth's rotation are of crucial importance, too.

In this lecture we put the main focus on normal hydrodynamics with simple source terms such as gravity. We will focus mostly on *grid-based methods for time-dependent fluid flows*. However, there will be a number of excursions away from that main line, so as to give a taste of the more complex versions of hydrodynamic study.

Literature

This lecture is inspired by, and partially follows, the following books:

- Randall J. LeVeque, ”Finite Volume Methods for Hyperbolic Problems” (Cambridge Texts in Applied Mathematics)
- Bodenheimer, Laughlin, Rozyczka and Yorke, ”Numerical Methods in Astrophysics: An Introduction” (Series in Astronomy and Astrophysics, Taylor and Francis)
- Toro, E.F. ”Riemann Solvers and Numerical Methods for Fluid Dynamics” (Springer Verlag)
- Collela and Puckett, ”Modern Numerical Methods for Fluid Flow”
- Liepmann, H. W. and Roshko, A., ”Elements of Gas Dynamics” (Dover Publications)
- Ferziger, J. H. and Peric, M, ”Computational Methods for Fluid Dynamics” (Springer Verlag)