Chapter 2

Geometric optics and the basic working of telescopes

Armed with our simple picture of "light as particles moving along straight lines" we can now investigate what happens if this light enters a mirror telescope. And with just a slight allusion to the wave nature of light we can also include the effect of lenses: The breaking of light on surfaces of changing refractive index. We will, however, postpone the more complex wave-related telescope phenomena until Chapter 3.

Literature:

The book by Smith, King & Wilkins, "Optics and Photonics, An Introduction" Second Edition, 2007, John Wiley and Sons, ISBN 9780470017845. An excellent introduction to the topic at a university student level.

Born & Wolf, "Principles of Optics", 1959, 2002, Press Syndicate of the Univeristy of Cambridge, ISBN 0521642221. A classic.

Léna, Lebrun & Mignard, "Observational Astrophysics", 2nd Edition, 1998, Springer-Berlin, ISBN 3540634827. Broad overview of the topic, though more for advanced reading.

Schroeder, "Astronomical Optics", 1999, ISBN 0126298106

2.1 A simple one-mirror telescope

The objective of a telescope is to project the source plane onto an image plane, where the light of each point on the source plane is bundeled onto a related point on the image plane. Between the source plane (which is in our case located at "infinity") and the image plane there is the optical system. The *aperture* of the optical system is the (usually circular) opening in the system through which the radiation enters the system. The larger the aperture, the more radiation can be collected, and thus the "deeper" one can image the sky. The term "aperture" can have many different meanings, depending on who uses the word, and in which context. We will come to this lateron.

In this section we wish to make a very simple telescope with just a single mirror. Let us put the mirror horizontally, in the $x-y$ plane, so that the normal of the mirror points upward. The shape of the mirror is determined by demanding that a bundle of parallal rays (i.e. emission from a source infinitely far away) going parallel to the mirror axis, are precisely bundled into a focal point at some height f above the mirror. This f is called the *focal length* of the mirror. From basic geometry we know that the shape of such a mirror has to be a parabola. So let us describe the shape of the mirror as

$$z = a(x^2 + y^2) \tag{2.1}$$

where x and y are the horizontal coordinates centered on the mirror center. The constant a determines the mirror bending and thus the location of the focal point. Again, from basic geometry we know that this focus point f is

$$f = \frac{1}{4a} \tag{2.2}$$

The point $(0, 0, f)$ is called the *primary focus* of the telescope (in this case, since we have just one mirror, it is not only the primary focus but the only focus).

So, by requiring the rays of a single point source on the sky to be bundled into a point, we have fully determined the shape of the mirror. The question then becomes: what about point sources that are slightly off-axis? Where will they be projected, and will the projection be as perfectly point-like as with the on-axis light? To zeroth order these off-axis point sources will all be projected onto their own foci on the focal plane: The plane determined by $z = f$ and arbitrary x and y. If we put a photographic plate (oldfashioned) or a CCD (modern) in this plane, we get an image of the sky. We can easily calculate where these foci of off-center point sources are. Let us first make a small coordinate system of the sky as seen by the telescope. Let us write the angles on the sky of our light sources with (n_x, n_y), where n_x and n_y are the x- and y-components of the unit vector $\vec{n}$ pointing toward the source. They are therefore celestial angles measured in radian. Since we assume that we will have a narrow field-of-view, i.e. $|n_x| \ll 1$ and $|n_y| \ll 1$, we can say that $n_z \simeq 1$. The point $(n_x, n_y) = (0, 0)$ corresponds to the point on the sky that lies on the axis of the mirror (the one that is per definition perfectly focused onto $(0, 0, f)$). We can now find the location (x_f, y_f, f) of the foci of any point (n_x, n_y), assuming them to lie in the focal plane, by following the ray that hits the mirror at $(0, 0, 0)$. We thus get

$$x_f = -n_x f \qquad\qquad y_f = -n_y f \tag{2.3}$$

Given that our detector typically has a technically limited spatial pixel size $\Delta x \times \Delta y$, it becomes clear that the larger we choose f the smaller objects on the sky we can spatially resolve. The relation between angular scale on the sky in arcsec and spatial scale on the focal plane in millimeter is:

$$\left(\frac{\delta x}{1\text{mm}}\right) = 4.85 \times 10^{-3} \left(\frac{\delta\theta}{1\text{arcsec}}\right) \left(\frac{f}{1\text{meter}}\right) \tag{2.4}$$

Given that CCDs have pixels the size of a few micrometers it is possible to put a camera at the prime focus of a modern telescope and still be able to resolve structures the size of about an arcsecond or less. In it, however, clear that a too long telescope would be unpractical. An often used quantity in telescope design is the *focal ratio* f/D where D is the diameter of the primary mirror. Ideally one would like to keep f/D not much larger than unity.

Let us return to the question: how well focused is the telescope away from the center of the focal plane? The easiest way to find out it to write a simple computer program that computes the reflection of a bundle of rays off the mirror at a non-zero incidence angle

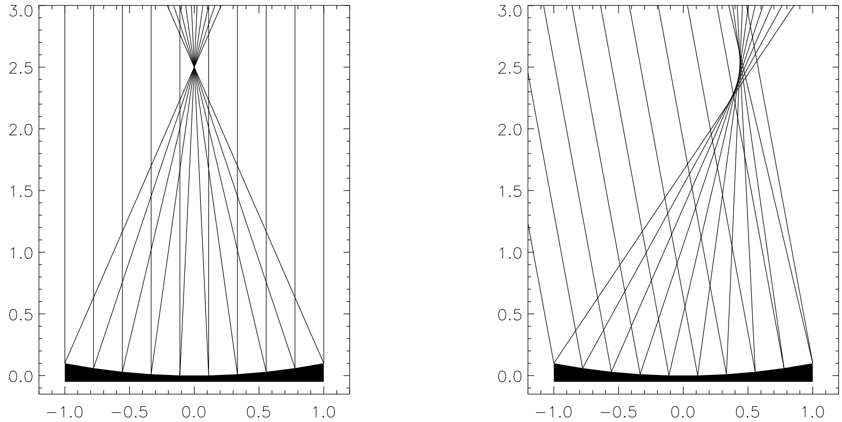

Figure 2.1: Results of a simple computation of ray reflection off a parabolic mirror with $a = 0.1$ and hence $f = 2.5$ for on-axis incident radiation (left) and off-axis incident radiation (right). The off-axis angle of incidence is 10 degrees.

and see if all rays cross each other in the same point or not. The results of a such a simple programming exercise (to be left to the reader) is shown in Fig. 2.1. One can see that the focusing for the off-axis source is not perfect. Of course in this case the situation is aggravated because we took such a large incidence angle. In any case: there is no point where the rays all come together, but there is a region where they are all reasonably close. This is called *coma*, which is one of the various possible forms of *optical aberration*.

The coma is stronger for points on the sky further away from the central pointing of the telescope (i.e. for which $\sqrt{n_x^2 + n_y^2}$ is larger). But given an off-axis point source, the coma will be stronger the larger the telescope is. The quantity of interest is the focal ratio f/D: the larger the focal ratio, the smaller the coma for a source at a given angle away from the axis.

2.2 Lenses

Lense-based telescopes (refractors) are no longer used in modern telescopes. This is because it is nearly impossible to build lenses of several meters across. But also, lenses have *chromatic aberration* effects: different wavelength act differently inside the telescope.

But for instruments installed in the foci of large telescopes lenses still play a large role. That is why we will discuss them here. Moreover, some of the stuff we discuss here we shall also use for the discussion of the Earth's atmosphere.

2.2.1 A lens as a dielectric medium

A lens is a dielectric medium that slows down light (or at least the *phase speed* of light) and thus makes the wavelength shorter than in vacuum. This leads to light breaking (refraction) at the lense surface, i.e. the changing of propagation of the direction of the light.

The ratio of the phase speed of light in the dielectric medium to the light speed in vacuum is called the *refractive index*:

$$n_\nu \equiv \frac{c}{v_{p,\nu}} \tag{2.5}$$

where $v_{p,\nu}$ is the phase speed of the light at frequency ν in the dielectric medium. For air at 1 bar at 0^oC at $\lambda = 589$nm we have $n = 1.00029$, while for glass (Pyrex) at the same wavelength we have $n = 1.470$. For vacuum it is by definition $n = 1$.

At a surface between two media with refrective indices n_1 and n_2, light will change its direction θ (measured such that $\theta = 0$ is normal to the plane) according to Snell's law:

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1} \tag{2.6}$$

which can be derived from simple wave front propagation arguments. Light entering a lens will thus move closer to the normal of the entrance surface ($n_2 > n_1$ implies $\theta_2 < \theta_1$). The reverse is true when it exits the lens.

Light will also be partly reflected on the surfaces, also as a result of the change in n. And if n has an imaginary component, then the medium will also absorb some of the radiation as it passes through it. We will not go further into the reflection or extinction here; we will focus on refraction.

One can now regard the working of a lens in various ways:

1. One can follow rays of light, how they bend on the surfaces of the lenses. This *ray-tracing* can be done for complex optical systems (many lenses and mirrors) using special ray-tracing software. Ray-tracing works well for cases where the wavelength of the radiation is very small compared to any of the lens diameters.

2. One can also regard a lens from a wave perspective, where you follow the wave fronts (and the reduction of the wavelength in lenses) as they pass through your system. For complex systems this is a much harder problem to solve, but it is valid also for cases where the wavelength is not so small compared to the lense diameters.

3. By combining the above two perspectives (the ray picture and the wave picture) one can derive an Euler-Lagrange picture: the ray always takes the route of smallest "effective distance" s which we define as $ds = n\ dl$, where l is the real distance along the ray. In mathematical terms the ray takes the path of $\delta \int n\ dl = 0$. This is *Fermat's principle*: light reaches a point in the shortest possible time.

4. Another wave-based picture of the working of a lens is that it is a *phase shifter*. Consider a plane wave moving in z direction, moving toward positive z. Now place a convex lens at $z = 0$ in the $x - y$ plane. Let us assume the lense has half-thickness h. At $z < -h$ the phases of the wave are constant in the $x - y$ plane. Once the wave passed the lens ($z > h$), the waves are no longer planar. If you look at the phases in the $x - y$ plane for $z > h$ you find that they differ from the phase at $(x, y) = (0, 0)$ in a way that is approximately proportional to $x^2 + y^2$ (a near spherically converging wave).

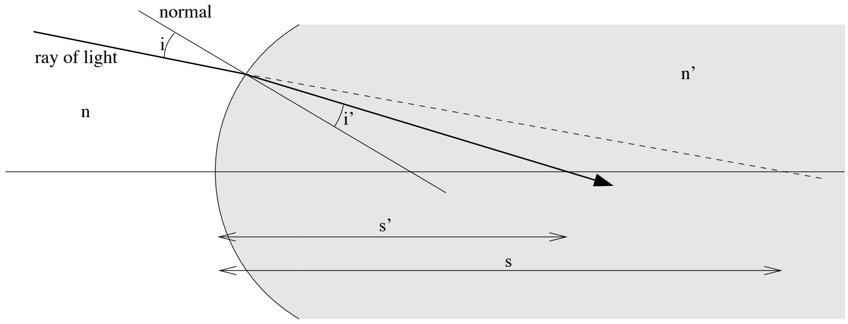

Figure 2.2: Refraction of light at a spherical interface between a medium of refractive index n (left) and a medium of larger refractive index n' (right).

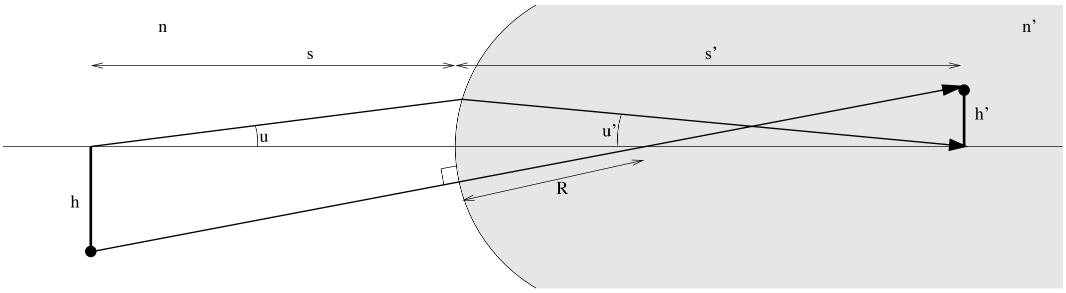

Figure 2.3: Same system as in Fig. 2.2, but now the case where the two conjugate points are on opposite sides of the spherical interface.

2.2.2 Refraction at a spherical interface

Let us look at a simple case first: that of a spherical interface between two media of different diffractive indices (Fig.2.2). The points at distances s and s' on the axis are called *conjugate points*. If $s = \infty$, then the point at s' is the *focal point*, or s' the *focal length*.

In the *paraxial approximation* we assume that all angles are small, so that one can approximate $\sin(i) \simeq i$ and $\sin(i') \simeq i'$. Snell's law thus becomes $ni = n'i'$. We define the *vertex* of the spherical optical surface as the point where this surface crosses the optical axis.

We can now have a look at the magnification of this "lens". This is depicted in Fig.2.3. In this case the conjugate points are on opposite sides of the interface. On the left we put some object. We start with a ray originating at the base of this object (i.e. at the optical axis), and find out how it refracts through the spherical interface. We thus find the point where it crosses the optical axis again. In the paraxial approximation the plane perpendicular to the optical axis and through this conjugate point defines the projection, i.e. where the points of our object get projected. Now take a ray starting from the top of the object, and make it go perpendicular through the spherical surface. This ray will not get refracted, because it is already perpendicular to the surface. By following the ray to the projection plane, we find where the top of the object is projected. We thus see that, in this particular case, the object gets smaller than the original.

The *transverse magnification* m is

$$m = \frac{h'}{h} = \frac{s' - R}{s - R} = \frac{ns'}{n's} \tag{2.7}$$

The *angular magnification* M is

$$M = \frac{\tan u'}{\tan u} = \frac{s}{s'} = \frac{n}{n'm} = \frac{nh}{n'h'} \tag{2.8}$$

The *Lagrange invariant* H is defined as

$$H = nh \tan u = n'h' \tan u' \tag{2.9}$$

In the paraxial approximation: $H = nhu = n'h'u'$. The *optical power* is defined as

$$P = \frac{n'}{s'} - \frac{n}{s} = \frac{n' - n}{R} = \frac{n'}{f'} = -\frac{n}{f} \tag{2.10}$$

Important note: You can use the same laws also for mirrors! Just put $n' = -1$. Exercise: Find the corresponding s and s' etc for a mirror.

2.2.3 Two surface optics: A one-lens or two-mirror system

In case of a lens we do not have just a single interface between two media, but two interfaces: an incoming and an outgoing. We thus have a two-surface optical system. The same is true for a two-mirror telescope. The power of such a system is

$$P = P_1 + P_2 - \frac{d}{n'} P_1 P_2 \tag{2.11}$$

where d is the distance between the two surfaces and n' is the refractive index of the material between the surfaces. If we put this lens in vacuum ($n = 1$), then $P = 1/f$, where f is the focal length of the lens. Using $n = 1$ we can then write this as

$$P = \frac{1}{f} = (n' - 1)\left[\frac{1}{R_1} - \frac{1}{R_2} + \frac{(n' - 1)d}{R_1 R_2 n'}\right] \tag{2.12}$$

For a thin lens ($d \simeq 0$) we thus have

$$P = \frac{1}{f} = P_1 + P_2 = (n' - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \tag{2.13}$$

Important note: The radii of curvature R_1 and R_2 can be both negative and positive. The sign convention is usually such that for the surface depicted in Fig. 2.3 the $R > 0$, while if the surface is bent the other way, then $R < 0$. This means that it the above formulae, if we have for instance a *biconvex* lens (see Fig. 2.4 for examples of lens types) with $|R_1| = |R_2| = R$, where R_1 is for the left surface and R_2 for the right surface, we have $R_1 = R$ and $R_2 = -R$. For $d \simeq 0$, then we get using Eq. (2.13)

$$f = \frac{R}{2(n' - 1)} \tag{2.14}$$

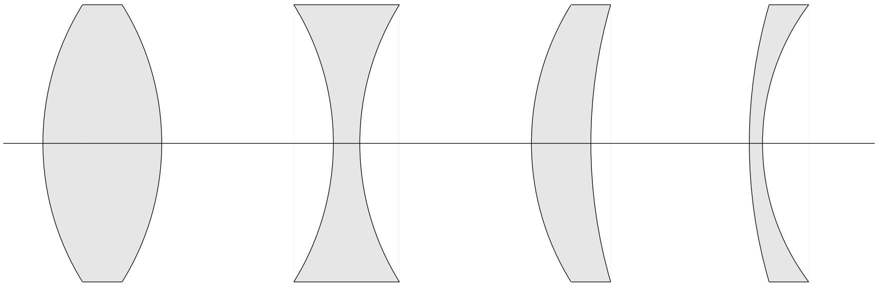

Figure 2.4: Some example shapes of lenses. From left to right: biconvex lens, biconcave lens, positive meniscus, negative meniscus.

2.3 Some more things...

2.3.1 Focal ratio

The *focal ratio* is the ratio of the focal length f and the diameter of the lens or mirror D:

$$F = \frac{f}{D} \tag{2.15}$$

Often this is written in the way $f/1.5$, meaning that $F = 1.5$. To be more precise, D is not just the diameter of the mirror or lens: It is the diameter of the *aperture*, i.e. the area that collects the light from outside. For a photocamera, the aperture can be set at will. If the aperture is taken to be small (small D), then the *exposure time* for obtaining a photo has to be taken long enough to collect sufficient light. The advantage for photography is that with a small aperture the *depth of field* of your photo is larger, i.e. both nearby and far-away objects appear acceptably sharp. The drawback is that if you want to photograph a fast-moving object, then these objects will smear out due to their motion. If you want to have the same *exposure* (i.e. the same total amount of photons hitting your film or CCD in a given image), but you want to reduce the smearing due to fast movement, you must instead choose a large aperture (large D), with the drawback of a reduced depth of field. It is for this reason that optical systems with small focal ratio are called *fast* and those with large focal ratio *slow*. In astronomy, where depth of field is not an issue, a large D is almost always preferred.

2.3.2 Fermat's principle

In optics, *Fermat's principle* (named after Pierre de Fermat, 1601-1665, a French lawyer and mathematician) says that the path a ray of light takes from point A to point B in space corresponds to paths of *stationary time*, in most cases the shortest time a photon takes. In vacuum this is evidently a straight line. But as we discussed above, light travels slower in a dielectric medium. This is why a ray of light can bend, while still keeping the time it takes for a photon to reach point B the shortest. Of course "short" is defined only in a local sense, because if there is a straight ray between points A and B that does not pass through the dielectric medium, then obviously there are two paths form A to B, one going through the dielectic medium and one not, where the one passing through the dielectric medium is clearly longer. That is why it is more correct to say that the time is *stationary*: slight variations of the path will not change its lengths in terms of the time it takes to go from A to B.

This is very similar to the Euler-Lagrange formalism. And a similar technique can be used to derive the shape of the ray between points A and B.