Exercises belonging to lecture
Observational Astronomy MKEP5 (SS 2011)

Sheet 2

20 “points” in total

1. **Focal length**

   (a) [1 pt] Derive the formula $x[\mu\text{m}] = 4.86 \cdot f[\text{m}] \cdot \theta['']$ for the size of the projection of an object of angular size θ on the focal plane of a telescope with focal length f.

   (b) [2 pt] We will see in a later chapter, that because of the wave-like nature of light, the maximum angular resolution of a telescope of diameter D is given by $\theta[\text{radian}] = 1.22\lambda/D$, where λ is the wavelength of the light. Show that for this resolution the following formula holds:

   $$\frac{x}{\lambda} \simeq \frac{f}{D} \tag{1}$$

   Interpret this result.

2. **Designing a simple hobby telescope**
   We have a digital camera with a CCD chip consisting of 1024x1024 pixels, sensitive to optical light ($\lambda = 0.55\mu$m). The chip is 2cm x 2cm large. We want to use it for astronomical observations at a site with typically 1” seeing[1].

   (a) [2 pt] What should be the focal length f of your telescope to ensure that a star creates a blob of 2x2 pixels on the CCD (seeing-limited observation)?

3. **Is bigger always better?**
   Suppose you want to make an image of the central region of the Andromeda Galaxy, which is an *extended* object even for small telescopes. Your friend at the Max-Planck-Institute for Astronomy allows you exactly 10 minutes of observing time on one of the two available telescopes. One has 30 cm diameter, the other 60 cm diameter. Both telescopes have focal ratio f/4. You have a CCD chip which you can put in the focal plane of either telescope. Of course you want to get nice image quality, i.e. you want to minimize the noise on each pixel,

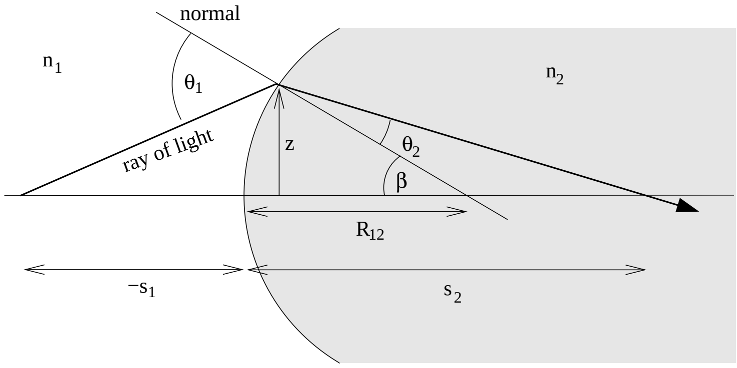
   (a) [2 pt] Argue, why it does not matter for the signal-to-noise ratio for each pixel whether you use the 30 cm or the 60 cm telescope.

   (b) [2 pt] Could you give a good reason to go for the 30 cm?

   (c) [2 pt] Could you give a good reason to go for the 60 cm?

[1] We will discuss the phenomenon of seeing in much more detail in a later chapter. For now: it is the smearing on the sky of a point source due to the atmospheric turbulence.

4. **The mathematics of lenses**
Consider a piece of glass of refractive index n_2 in a medium with refractive index n_1. The piece of glass has a spherical side to the left, with radius of curvature R_{12}. Consider a ray passing through:



Light emitted from a point at location at $x = s_1$ is refocussed at point $x = s_2$. Here $s_1 < 0$ and $s_2 > R_{12} > 0$, where x is the horizontal coordinate in the figure. In the paraxial approximation we focus on light rays for which $|z| \ll |R_{12}|$, $|z| \ll |s_1|$ and $|z| \ll |s_s|$, where z is the vertical coordinate of the point at which the ray enters the glass (see figure). In this approximation the angles θ_1 and θ_2 are small, so that $\sin\theta_1 = \tan\theta_1 = \theta_1$ and likewise for θ_2. Snell's law becomes $n_1\theta_1 = n_2\theta_2$.

(a) [3 pt] Prove, in the paraxial approximation, the following relation:

$$\frac{n_2}{s_2} - \frac{n_1}{s_1} = \frac{n_2 - n_1}{R_{12}} \tag{2}$$

(b) [3 pt] Show that the focal length on the left side (f_1) and the right side (f_2) are respectively:

$$f_1 = \frac{n_1 R_{12}}{n_1 - n_2} \qquad \text{and} \qquad f_2 = \frac{n_2 R_{12}}{n_2 - n_1} \tag{3}$$

Note: $f_1 < 0$.

(c) [3 pt] The "Power" P_{12} is defined as

$$P_{12} = \frac{n_2}{f_2} = -\frac{n_1}{f_1} \tag{4}$$

Now add another curved edge to the piece of glass, with power P_{23}, this time to the right side, at a distance d from the left edge. Let us take $d \ll |R_{12}|$ and $d \ll |R_{23}|$, i.e. small enough that we can ignore it. Also assume that the external medium has $n_3 = n_1 = 1$ (vacuum), and call the refractive index of the glass simply n. Derive the *thin lens equation*, or in other words: the rule for the summing of powers:

$$P = P_{12} + P_{23} \tag{5}$$

where $P \equiv 1/f$ with f the focal length of the total piece of glass (i.e. lens).