Exercises belonging to lecture Observational Astronomy MKEP5 (SS 2011)

Sheet 4

20 “points” in total

1. **Phase shifter**
   In the lecture on diffraction, a lens was introduced as a “phase shifter” into the aperture of the camera obscura. Here we will play a bit more with the concept of phase shifters. We use, as in the lecture, the coordinates x' and y' as the coordinates in the aperture plane.

   (a) [2 pt] Suppose the phase shift $\Delta\phi(x', y')$ is given by

   $$\Delta\phi(x', y') = \frac{2\pi x'}{L} \tag{1}$$

   for some value of L. If a parallel wave front with wavelength $\lambda \ll L$ falls perpendicularly onto the phase shifter, and the aperture is much larger than the wavelength, can you tell what shape the wave crests behind the phase shifter have? Ignore the diffraction on the edges of the phase shifter; focus on the waves just behind the phase shifter. Be quantitative.

   (b) [3 pt] Let us now make such a phase shifter from glass. Glass has a refractive index $n = 1.5$ at $\lambda = 0.5\mu$m (visible light). We make a very thin plate of glass with width W_0 at the center, but with a gradient in the width given by

   $$W(x', y') = W_0 + W_1 x' \tag{2}$$

   where W_1 sets the gradient. How are L (above) and W_0, W_1 (here) related? Give an expression.

2. **Direct imaging of exoplanets**
   One of the goals of the E-ELT, the 42-meter diameter “European Extremely Large Telescope” that is planned to be built in Chile, is the direct detection of exoplanets very close to the host star. Let us for simplicity approximate the mirror shape with a circular disc of 42 meter diameter (and without hole, nor support structures). Suppose that the atmospheric seeing can be entirely removed by a perfect adaptive optics system[1].

   (a) [2 pt] We observe a sun-like star at a distance of 30 pc. It has an Earth-like planet at 1 AU orbital radius. Does the E-ELT have sufficient spatial resolution at $\lambda = 0.5\mu$m to observe the planet (ignoring sensitivity issues for now; just focusing on spatial resolution)?

[1] See later in the lecture for *real* adaptive optics systems which are, of course, far from perfect

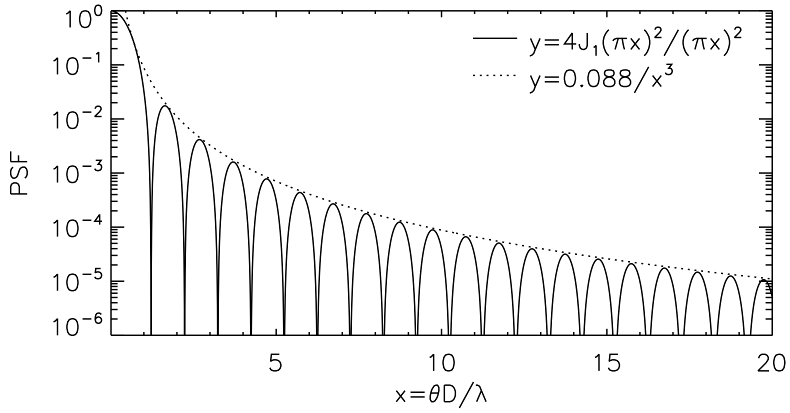

Figure 1: The Airy function (solid line) and its powerlaw approximation (dotted line).

One problem is that the PSF of the bright star will overwhelm the flimsy light of the planet. We will estimate how severe this is. The Airy function is plotted in Fig. 1 (solid curve), normalized in such a way that it is 1 at $x = 0$. To make things a bit simpler we approximate the Airy function far away from the central peak with $4J_1(\pi x)^2/(\pi x)^2 \simeq 0.088/x^3$. Both functions (the real PSF and the approximation) are shown in Fig. 1.

(b) [2 pt] Assume the planet is covered in snow with 100% reflectivity in the optical. It's radius is $R_{\rm planet} = R_{\rm Earth} = 6366$ km, and its circular orbital radius is $a_{\rm planet} = 1$ AU $= 1.5 \times 10^8$ km. Take the planet at its largest angular distance from the star. What is the expected luminosity ratio $L_{\rm planet}/L_{\rm star}$?

(c) [2 pt] Using the approximation of the Airy function given above, how much would the star PSF at the location of the planet be brighter than the planet?

(d) [2 pt] The real PSF has many zero-points as one can see in Fig. 1. It would therefore, in principle, be possible to get much better contrast between the planet and the stellar PSF if the planet happens to be at such a zero point. Speculate on the feasibility of this.

3. **Airy disk: Scaling relations**
In the script we derived the flux F that falls on the focal plane of a simple camera obscura[2]:

$$F(x, y) = E_0^* E_0 b^2 \left[\frac{J_1\left(\frac{2\pi b}{\lambda z_1}\sqrt{x^2+y^2}\right)}{\sqrt{x^2+y^2}} \right]^2 \tag{3}$$

where b is the radius of the aperture.

(a) [2 pt] If we make the radius of the aperture twice as large, by which factor does the flux F at the peak of the PSF change, and why?

[2]Note: in the script there was an error in this formula, that has been fixed.

(b) [1 pt] If we make the lens twice as weak, and thus have the focal plane twice as far from the aperture, by which factor does the flux F at the peak of the PSF change, and why?

4. **Gaussian aperture**

The aperture of a telescope is usually well described by a circular hole in a black screen. But suppose we replace this screen+hole with a glass plate with a transparency $T(x', y')$ given by the following Gauß-function:

$$T(x', y') = \exp\left(-\frac{(x')^2 + (y')^2}{a^2}\right) \tag{4}$$

where a is a value that sets the width of the Gaussian. A transparency of 1 means that the light goes through unhindered, while a transparency of 0 means that the light is 100% blocked. Behind that aperture we place a lens with focal length f. The lens is much larger than a, so the finite size of the lense plays no role. Also $f \gg d$.

(a) [1 pt] What does a transparency of $T(x', y')$ mean for the electric field (as opposed to the light intensity)?

(b) [3 pt] Give the shape and width of the Point Spread Function at the focal plane.