Exercises belonging to lecture Observational Astronomy MKEP5 (SS 2011)

Sheet 5

20 "points" in total

1. **Not-entirely-drunk man's walk**
   A standard example in statistics is the "random walk" motion, sometimes called "drunk man's walk". The idea is that a totally drunken guy walks every time 1 step (=1 meter) in a random direction, each step being again a new random direction. As we know, the expectation value of its position after N steps is zero:

   $$\langle \begin{pmatrix} x \\ y \end{pmatrix} \rangle = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{1}$$

   even though the expectation value of the *distance* from the starting point $\langle r \rangle = \sqrt{N}$. It basically says that the drunkard has not systematically moved in any direction, he has only randomly drifted away in an arbitrary direction.

   Now let's look at a guy who is *almost sober*, but still has some uncertainty in his direction. Let us define the direction in which he walks as ϕ such that the step he takes is given by:

   $$\begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix} = \begin{pmatrix} \cos\phi \\ \sin\phi \end{pmatrix} \tag{2}$$

   The probability distribution function for him taking a step in direction ϕ is given by:

   $$P_\phi(a) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{a^2}{2\sigma^2}\right) \tag{3}$$

   (a) [4 pt] What is the expectation function for the distance the guy walks in x-direction after N steps. Hint: Take $N \gg 1$.

2. **Seeing from a layer**
   In the lecture we have seen qualitatively how a wavefront distortion can make the star move from it's real position. Let us find some numbers. Suppose we have an atmosphere that is for the most part quiet, i.e. without temperature variations (other than the usual atmospheric temperature gradient). There is, however, a layer of air from 1 km to 1.001 km (i.e. 1 meter thick) which consists of pockets of air of horizontal size L with different temperatures from their surroundings. Approximate this with

   $$T(x,y) \simeq 300 + 1\ \text{Kelvin}\ \sin\left(\frac{2\pi x}{L}\right) \tag{4}$$

   (we ignore the vertical temperature gradient here). The layer moves with a speed of $v = 10$ m/s in $x - direction$. The mean density of the air $\langle\rho\rangle = 1.3 \times 10{-}3$ g/cm^3. The star we observe is at zenith.

(a) [3 pt] Eq. (4) gives the temperature variations in the layer. Give the corresponding density variations (give the formula).

(b) [2 pt] Give the corresponding variations in the refractive index.

(c) [2 pt] What is the maximum angular deviation a star will get due to this layer of turbulence in arcseconds?

(d) [2 pt] Give the variations in the phase of a wave of wavelength λ once it has passed the layer.

(e) [1 pt] For which wavelength is this phase shift between the maximum and minimum of the temperature fluctuation (i.e. at $x = 0.25\,L$ and $x = 0.75\,L$) exactly 2π?

(f) [2 pt] Take a telescope with diameter $D = L$, and observe at the wavelength we just derived. Compare the diffraction limited angular resolution with that of the seeing.

3. **Autocorrelation and Structure function**
We have a function

$$g(t) = \begin{cases} 1 & \text{for} \quad |t| \leq 1 \\ 0 & \text{for} \quad |t| > 1 \end{cases} \tag{5}$$

(a) [2 pt] Give the autocorrelation function $B_g(\tau)$

(b) [2 pt] Give the structure function $D_g(\tau)$