Appendix C

Noise, error propagation

Observational astronomy involves measuring signals with noise. When we make a finite-duration measurement we therefore always have some degree of noise on our signal. We may have also systematic errors, but in this appendix we will focus on stochastic errors (noise) instead, because they can be treated in a general way.

The "error" or uncertainty of a particular measurement due to noise can be quantified with a *standard deviation* σ. Suppose we have, of some quantity f, N measurements. The average $\langle f\rangle$ and standard deviation σ_f are then

$$\langle f\rangle = \frac{1}{N}\sum_{i=1}^{N} f_i \qquad \sigma_f^2 = \frac{1}{N}\sum_{i=1}^{N}(f_i - \langle f\rangle)^2 \tag{C.1}$$

If we have two quantities, f and g, with standard deviations σ_f and σ_g, and we wish to know the standard deviation of a third quantity q that is constructed out of f and g, then we can write

$$\sigma_q^2 = \left(\frac{\partial q}{\partial f}\right)^2 \sigma_f^2 + \left(\frac{\partial q}{\partial g}\right)^2 \sigma_g^2 \tag{C.2}$$

This is called *error propagation* from f and g to a new quantity q. Note, it obviously also holds of we only have a function q of f only, and, equally obviously, with arbitrary number of functions.

Example: $q = f+g$ gives $\sigma_q^2 = \sigma_f^2+\sigma_g^2$. Another example: $q = fg$ gives $\sigma_q^2 = g^2\sigma_f^2+f^2\sigma_g^2$.