Exercises for Radiative Transfer in Astrophysics (SS2012)

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Exercise sheet 2

The Formal Transfer Equation

1. **Layer above a semi-infinite medium**
   Consider the following 1-D plane-parallel radiative transfer problem. We have an optically thick semi-infinite gaseous medium extending from $z = -\infty$ to $z = 0$. The temperature of that medium is 6000 K. Above that is a layer of homogeneous density $\rho = 1 \times 10^{-7}$ gram cm^{-3} extending from $z = 0$ to $z = 5 \times 10^{7}$ cm. Its temperature is 5000 K. The opacity of the material in the layer is

   $$\kappa_\nu = 10^{11} \frac{\gamma}{(\nu - \nu_0)^2 + \gamma^2} \,\mathrm{cm}^2\,\mathrm{gram}^{-1} \tag{1}$$

   where $\nu_0 = 3 \times 10^{14}$ Hz.

   (a) Make a plot of the shape of the absorption line for the case $\gamma = 3 \times 10^{10}$ Hz, as seen from straight above ($i = 0$, i.e. $\mu = 1$). Take $(\nu/\nu_0 - 1)$ as the x-axis. Make sure to choose the right range of frequencies around ν_0 to see most of the absorption profile. If everything went right, you should see that the absorption line saturates at the bottom.

   (b) Now do the same for $\gamma = 3 \times 10^{11}$ Hz. If everything went right the saturation is minor in this case.

2. **A simple stellar atmosphere model**
   Consider the following simple model of a stellar atmosphere. As in the previous exercise we assume that for $z < 0$ we have an optically thick semi-infinite gaseous medium with temperature 6000 K. Between $z = 0$ and $z = 10^{8}$ cm we have the density going as:

   $$\rho(z) = 3 \times 10^{-7} \exp\left(-\frac{z}{10^7\,\mathrm{cm}}\right) \mathrm{gram\,cm}^{-3} \tag{2}$$

   and zero above that. The temperature (for $z > 0$) goes as

   $$T(z) = 6000 - 1000 \frac{z}{10^8\,\mathrm{cm}} \,\mathrm{K} \tag{3}$$

   For the opacity we still have Eq. (1) with $\gamma = 3 \times 10^{10}$ Hz.

   (a) Write a compute program that integrates the formal transfer equation for this problem, assuming only thermal absorption and emission (i.e. no scattering). You can use the first order quadrature formula from the lecture notes.

(b) Make a plot of the intensity I as a function of z for $\mu = 1$ at $\nu = \nu_0$. You should see that the intensity goes down with z, but then levels off. Does it level off at $B_\nu(5000K)$? If yes, why? If not, why not – and what causes the leveling off?

(c) Plot the spectrum as seen from straight above ($i = 0$).

(d) Overplot the spectrum as seen from under an angle: $i = 60^o$. Why is the absorption at $\nu = \nu_0$ deeper in this case?

For all exercises, please always do the following:

- Make an electronic document (DOC or PDF) which includes your text concerning the exercises, as well as figures belonging to it.
- Upload your document *and your computer program* to the Moodle.