Exercises for
Radiative Transfer in Astrophysics (SS2012)

Cornelis Dullemond

Exercise sheet 4

Accelerated Lambda Iteration & Ng-acceleration

1. **Multiple isotropic scattering in a 1-D plane-parallel atmosphere**
   Let us take the same atmosphere as in exercise 3. But this time we will use the ALI and Ng techniques. You do not need to program these yourself. Instead, please download the twostream.f90 program from the website of the lecture or from the Moodle and compile it with:

   ```
   gfortran twostream.f90
   ```

   It will produce a file called `a.out` which is the executable. You can start it by typing:

   ```
   ./a.out
   ```

   The program will ask you several questions which allow you to specify the photon destruction probability ϵ, as well as several settings for the LI or ALI iteration. Then it will perform the iteration and write the result to a file called `s_final.out`. It will also write all the intermediate results of the iteration in a file called `s_iter.out`, and it will also compute the "true" two-stream solution by making use of the equivalence between two-stream and diffusion (NOTE: this is only the "true" solution within the framework of the two-stream approximation!). That solution is written to `s_diff.out`. You can use that solution to compute the degree of convergence of the (A)LI method at each iteration step n:

   $$E^n = \max_{i=1}^{N_z} \left(\frac{|S_i^n - S_{\mathrm{solu},i}|}{S_{\mathrm{solu,i}}} \right) \tag{4}$$

   where $S_{\mathrm{solu}} = S_{\mathrm{diff}}$ is the "real" solution.

   (a) Use $N_{\mathrm{iter}} = 100$, order=3, $\epsilon = 1d-2$. Try out the LI, ALI-local, ALI-tridiag methods without and with Ng acceleration. Make a plot of E^n as a function of n so that you can see the differences in convergence of the various methods. Please take the y-axis logarithmic between 10^{-8} and 1, so that you can see the full convergence.

   (b) Do the same for $\epsilon = 10^{-4}$.

   Normally you do *not* know ahead of time what S_{solu} is (otherwise, what's the point of doing the iteration!?). So what you normally plot is the convergence progress according to:

   $$D^n = \max_{i=1}^{N_z} \left(\frac{|S_i^n - S_i^{n-1}|}{S_i^{n-1}} \right) \tag{5}$$

   (c) Do the same as above, but now with D^n instead of E^n.

(d) Interpret what you see. In particular the spikes in D every few iteration steps.

Now let us look at the importance of the order of integration.

(e) Use $N_{\text{iter}} = 100$, ALI-tridiag, Ng-acceleration, order=2, and plot E^n as a function of n for $\epsilon = 10^{-2}$, $\epsilon = 10^{-4}$ and $\epsilon = 10^{-6}$. When does the second order integeration no longer converge to appreciable accuracy?

(f) Do the same for oder=1.

For all exercises, please always do the following:

- Make an electronic document (DOC or PDF) which includes your text concerning the exercises, as well as figures belonging to it.
- Upload your document *and your computer program* to the Moodle.