Exercises for
Radiative Transfer in Astrophysics (SS2013)

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Exercise sheet 6

Spherical circumstellar dusty envelope model (part II)

We continue with the model of an optically thick spherically symmetric dusty cloud around a star of exercise sheet 5.

1. **Optical depth of the envelope**
   In this exercise we are going to calculate the radial optical depth of the envelope (from the star out to infinity) for the case of `rhodust0=1d-18` (i.e. $\rho_{\mathrm{dust},0} = 10^{-18}$ gram/cm^3, the normal case). Let us first do this at a wavelength of around $\lambda \simeq 0.5$ μm. Do this with the following steps:

   (a) Look in the table in the file `dustkappa_silicate.inp` and find the wavelength point closest to $\lambda \simeq 0.5$ μm. Take the corresponding κ_{abs} as the opacity. If you want to do an even better job (voluntary): take the two closest wavelength points and do a linear interpolation of κ_{abs}.

   (b) Write down the analytic form of the integral of the optical depth at $\lambda \simeq 0.5$ μm for this powerlaw density distribution, and solve it. You will obtain an expression for the optical depth at this wavelength, $\tau_{0.5}$, as a function of $\rho_{\mathrm{dust},0}$. By inserting the value of $\rho_{\mathrm{dust},0} = 10^{-18}$ you get the optical depth. This should, for $\rho_{\mathrm{dust},0} = 10^{-18}$ gram/cm^3, give a value around $\tau_{0.5} \simeq 580$.

   If this were a grey envelope, then the optical depth of 580 would mean that radiation would have a very hard time to escape. A radiation package would get absorbed and re-emitted on average 580^2 times (note that an absorption + re-emission event is in some sense like a scattering event, so we can use what we've learned about the number of scatterings at some optical depth). However, the opacity table is definitely non-grey. Let us take this into account now.

   (c) The dust in the inner edge of the envelope has a temperature of about 70 K. The peak of the function $\nu B_\nu(T)$ is located at a frequency $\nu = 4kT/h$. What is the opacity at that frequency? If you take the opacity at that frequency, show that the optical depth is roughly around 6. How many absorption + re-emission events do you expect?

   (d) RADMC-3D, however, writes that on average the photons experience even fewer events (roughly 11). Any idea why? Tip: The temperatures at larger radii than the inner edge are lower than 70 K.

2. **The SED**

(a) Use `RADMC-3D` to create the SED of this envelope covering all wavelength of interest (from the UV to the millimeter) and make a plot of this. The observer is at a distance of 100 parsec. Note: since RADMC-3D is optimized for 3-D transfer, it is not particularly fast for 1-D problems, so you may need to wait a few minutes before it is finished. Tip: Find in the manual of RADMC-3D how to make an SED. The file `spectrum.out` that RADMC-3D produces contains some header lines and then two columns. The first one is the wavelength, the second one is the flux (in erg/cm^2/s/Hz) of the object when the observer is at a distance of 1 parsec.

(b) Do the same, but now for an envelope with $\rho_{\mathrm{dust},0} = 10^{-21}$ gram/cm^3 (i.e. a 1000x lower optical depth). Explain the difference with the previous one.